C.P. No. 170
(15,349)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Aerodynamic Derivatives for a Delta Wing Oscillating in Elastic Modes

By

D. L. Woodcock, M.A.

LONDON: HER MAJESTY'S STATIONERY OFFICE

1954

Price 3s. 0d. net

C.P. 170

Report No. Structures 132

July, 1952

ROYAL AIRCRAFT ESTABLISHMENT

Aerodynamic Derivatives for a Delta Wing Oscillating in Elastic Modes

by

D.L. Woodcock, M.A.

SUMMARY

Aerodynamic derivatives are given for a delta wing of aspect ratio 3 and 90° apex angle oscillating with symmetric elastic modes in incompressible inviscid flow. They have been determined by the lattice method of W.P. Jones, using the values of the downwash calculated by D.E. Lehrian when obtaining aerodynamic derivatives for the same delta wing oscillating in rigid wing modes.

The results for several modes of the form $|\eta|^n$ are given both as local derivatives and also as equivalent constant derivatives, that is derivatives invariable with spanwise position, with the virtual inertias included in the aerodynamic stiffness derivatives. Derivatives for other modes can be obtained either from these or from the values of the reciprocal $\bar{W}^{-1}$ of the downwash matrix, which also are tabulated.

LIST OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

1 Introduction

The use, in flutter calculations, of aerodynamic derivatives obtained from two-dimensional theory modified by simple reduction factors has proved reasonably satisfactory for wings of large or moderate aspect ratio and small taper. For highly tapered wings of small aspect ratio, however, it is expected that the aerodynamic derivatives will have to be determined on a more accurate three-dimensional basis. W.P. Jones[1] has suggested a method for calculating these derivatives for incompressible flow and D.E. Lehrian[2] has applied it to a delta wing oscillating in rigid wing modes. For wing flutter investigations, derivatives will also be required for modes of elastic deformation of the wing. This report describes a limited application of the method to the calculation of such derivatives.

In the work here described, aerodynamic derivatives have been calculated for a cropped delta wing having aspect ratio 3 and 90° apex angle, and oscillating with symmetric elastic modes in incompressible flow. The planform is the same as that considered by D.E. Lehrian[2] and was chosen so that some of the intermediate results calculated by her could be used. The derivatives have been obtained for several modes of the form $|\eta|^n$, where η is a non-dimensional spanwise co-ordinate. They are given both as local derivatives and as equivalent constant derivatives, that is derivatives which are chosen to be constant over the span and to give the correct generalised aerodynamic forces. The virtual inertias are included in the aerodynamic stiffness derivatives. The derivatives are compared, for general interest, with constant derivatives obtained on the basis of two-dimensional theory.

Derivatives for other modes can be obtained either by approximating to them, if possible, in terms of the above mentioned modes and then using the derivatives that have been calculated, or else by using the formulae (given later) which express the derivatives in terms of the reciprocal $\bar{W}^{-1}$ of the downwash matrix. Values of $\bar{W}^{-1}$ are given for three values of the frequency parameter.

2 Theoretical analysis

2.1 Method

Any point on the wing is described by the two non-dimensional co-ordinates η and θ, such that the distance (x) of the point aft of the mid chord axis is $-\frac{c}{2}\cos\theta$, and the distance spanwise from the aircraft centre line is $s\eta$, where c is the local wing chord and s is the wing semi-span. Thus

$$\eta = 0 \text{ at the centre line}$$

$$= \pm 1 \text{ at the wing tips}$$

$$\theta = 0 \text{ at the leading edge}$$

$$= \pi \text{ at the trailing edge}$$

The wing is considered to be a flat plate. The vertical displacement of any point on it is denoted by $ze^{\lambda t}$, where z is a function of η and θ and $\lambda = i\omega$ where ω is the circular frequency. If $Ke^{\lambda t}$ is the discontinuity in the velocity potential field over the wing and the wake, i.e.

$$Ke^{\lambda t} = \phi_a - \phi_b \qquad (1)$$

where the suffices a, b refer to flow above and below the wing respectively, then the corresponding pressure difference is

$$p_a - p_b = -\rho \frac{d}{dt}(Ke^{\lambda t}) = (\text{say}) -\rho V \Gamma e^{\lambda t}, \qquad (2)$$

where $\Gamma e^{\lambda t}$, the bound vorticity, is zero everywhere except on the wing. The discontinuity in the velocity potential can be represented by a doublet distribution over the wing and the wake of strength $Ke^{\lambda t}$ per unit area. On the basis of the results of two dimensional theory[1] it was assumed that this doublet distribution was of the form

$$K = cV\left\{(S_o' + S_o'') \sum_{m=1}^{\infty} C_{om} A_m + \sum_{n=1}^{\infty} S_n \sum_{m=1}^{\infty} C_{nm} A_m\right\} \qquad (3)$$

where the S_n's are the functions of θ and the frequency parameter which arise in the two-dimensional theory[1], the C_{nm}'s are arbitrary constants, and the A_m's are functions of η given by

$$A_m = \frac{s}{c} T_m = \frac{s}{c} \eta^{m-1}\sqrt{1-\eta^2} \qquad (4)$$

For symmetric motion, only the symmetric T_m functions (i.e. m odd) are used. The corresponding distribution of bound vorticity is (omitting the $e^{\lambda t}$)

$$\Gamma = V\left\{(\Gamma_o' + \Gamma_o'') \sum_{m=1}^{\infty} C_{om} A_m + \sum_{n=1}^{\infty} \Gamma_n \sum_{m=1}^{\infty} C_{nm} A_m\right\} \qquad (5)$$

where the Γ_n's are the functions given in reference 1; and the normal induced velocity (or downwash velocity) is $We^{\lambda t}$, where

$$W = V\left\{\sum_{m=1}^{\infty} C_{om} (W_{om}' + W_{om}'') + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C_{nm} W_{nm}\right\} \qquad (6)$$

i.e. W_{nm} is the downwash velocity due to a doublet distribution $c\, S_n A_m$, etc.

The downwash W must satisfy the condition

$$W = V\frac{\partial z}{\partial x} + \lambda z \qquad (7)$$

at all points on the wing.

2.2 Application

To make the calculations tractable, the series (3), (5) and (6) were curtailed after a finite number of terms, and the condition (7) was satisfied only at a finite number of points called collocation points. The terms omitted were those for n greater than one or for m greater than five. The six collocation points thus required to determine the coefficients C_{nm} were taken as (η_1, θ_1), (η_2, θ_1), (η_3, θ_1), (η_1, θ_2), (η_2, θ_2) and (η_3, θ_2) where

$$\eta_1 = 0.2$$
$$\eta_2 = 0.6$$
$$\eta_3 = 0.8 \tag{8}$$

and

$$\theta_1 = \pi/2$$
$$\theta_2 = \cos^{-1}(-\,^2/3) \tag{9}$$

The method which Miss Lehrian used to obtain the W_{nm}'s at these collocation points is described in references 1 and 2.

When there is no distortion of chordwise sections the downward displacement of any point on the wing can be written

$$ze^{\lambda t} = e^{\lambda t} \sum_i q_i \left(s \,.\, f_i(\eta) - \frac{c}{2} \cos\theta \,.\, F_i(\eta) \right) \tag{10}$$

where the q_i are the co-ordinates of the degrees of freedom. The function f_i defines the mode of normal translation along the mid-chord axis, and F_i the change of incidence of fore-and-aft sections. Equation (7) then becomes

$$W = V \sum_i \left\{ u(f_i(\eta) - \frac{c}{2s} \cos\theta \,.\, F_i(\eta)) + F_i(\eta) \right\} q_i \tag{11}$$

where

$$\mu = \frac{\lambda s}{V} \tag{12}$$

We write

$$W_o(\theta_1) = \begin{bmatrix} W_{o1}(\eta_1,\theta_1) & W_{o1}(\eta_2,\theta_1) & W_{o1}(\eta_3,\theta_1) \\ W_{o3}(\eta_1,\theta_1) & W_{o3}(\eta_2,\theta_1) & W_{o3}(\eta_3,\theta_1) \\ W_{o5}(\eta_1,\theta_1) & W_{o5}(\eta_2,\theta_1) & W_{o5}(\eta_3,\theta_1) \end{bmatrix} \tag{13}$$

and similar expressions for $W_o(\theta_2)$, $W_1(\theta_1)$ and $W_1(\theta_2)$

where

$$W_{om} = W'_{om} + W''_{om}$$

and also let

$$C_o = \begin{bmatrix} C_{o1} & C_{o3} & C_{o5} \end{bmatrix} \tag{14}$$

and

$$C_1 = \begin{bmatrix} C_{11} & C_{13} & C_{15} \end{bmatrix} \tag{15}$$

Then by equating W from (6) and (11) at the collocation points, we obtain the equation

$$C_o W_o(\theta_1) + C_1 W_1(\theta_1) = \sum_i q_i \left\{ \mu \left(f_i - \frac{\cos \theta_1}{2} F_i H\right) + F_i \right\} \quad (16)$$

and
$$C_o W_o(\theta_2) + C_1 W_1(\theta_2) = \sum_i q_i \left\{ \mu \left(f_i - \frac{\cos \theta_2}{2} F_i H\right) + F_i \right\} \quad (17)$$

where
$$f_i = \begin{bmatrix} f_i(\eta_1) & f_i(\eta_2) & f_i(\eta_3) \end{bmatrix} \quad (18)$$

$$F_i = \begin{bmatrix} F_i(\eta_1) & F_i(\eta_2) & F_i(\eta_3) \end{bmatrix} \quad (19)$$

and
$$H = \begin{bmatrix} \frac{c}{s}(\eta_1) & 0 & 0 \\ 0 & \frac{c}{s}(\eta_2) & 0 \\ 0 & 0 & \frac{c}{s}(\eta_3) \end{bmatrix} \quad (20)$$

Equations (16) and (17) can be combined in one equation by putting

$$\bar{C} = \begin{bmatrix} C_o & C_1 \end{bmatrix} \quad (21)$$

$$\bar{W} = \begin{bmatrix} W_o(\theta_1) & W_o(\theta_2) \\ W_1(\theta_1) & W_1(\theta_2) \end{bmatrix} \quad (22)$$

$$\bar{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix} \quad (23)$$

$$\bar{f}_i = \begin{bmatrix} f_i & f_i \end{bmatrix} \quad (24)$$

$$\bar{F}_i = \begin{bmatrix} F_i & F_i \end{bmatrix} \quad (25)$$

and
$$\Theta = \begin{bmatrix} \cos \theta_1 \cdot I_3 & 0 \\ 0 & \cos \theta_2 \cdot I_3 \end{bmatrix} \quad (26)$$

where I_3 is the third order unit matrix. The combined equation is then

$$\bar{C}\,\bar{W} = \sum_i q_i \left\{ \mu \bar{f}_i - \frac{\mu}{2} \bar{F}_i \bar{H} \Theta + \bar{F}_i \right\} \quad (27)$$

from which the unknown $\bar{C}$ can be found.

2.3 Equivalent constant derivatives

The virtual work in small displacements δq_i for one wing is, from equations (2), (5) and (10), (omitting the $e^{2\lambda t}$)

$$\sum_i Q_i \cdot \delta q_i = -\rho V s^2 \int_0^1 \int_0^\pi \Gamma \frac{1}{2} \frac{c}{s} \sin\theta \cdot \delta z \cdot d\theta \cdot d\eta$$

$$= -\rho V^2 s^3 \sum_i \delta q_i \int_0^1 \int_0^\pi \left(\Gamma_o \sum_m C_{om} A_m + \Gamma_1 \sum_m C_{1m} A_m\right) \tag{28}$$

$$\times \frac{1}{2}\frac{c}{s}\left(f_i(\eta) \sin\theta - \frac{1}{2}\frac{c}{s} F_i(\eta) \sin\theta \cos\theta\right) d\theta . d\eta$$

Q_i being, by definition, the generalised aerodynamic force associated with the co-ordinate q_i.

But

$$\Gamma_o = \Gamma_o' + \Gamma_o'' = 2\left(C \cot\frac{\theta}{2} + \frac{\lambda c}{2V} \sin\theta\right) \tag{29}$$

and

$$\Gamma_1 = -2\sin\theta + \cot\frac{\theta}{2} + \frac{\lambda c}{2V}\left(\sin\theta + \frac{\sin 2\theta}{2}\right) \tag{30}$$

(cf reference 1) where C is the usual two dimensional lift function, i.e.

$$C = K_1\left(\frac{\lambda c}{2V}\right) \Bigg/ \left\{K_o'\left(\frac{\lambda c}{2V}\right) + K_1\left(\frac{\lambda c}{2V}\right)\right\} \tag{31}$$

where K_o, K_1 are modified Bessel Functions of the second kind.

Thus

$$\int_0^\pi \Gamma_o \sin\theta \, d\theta = \left(2C + \frac{\mu}{2}\frac{c}{s}\right)\pi$$

$$\int_0^\pi \Gamma_1 \sin\theta \, d\theta = \frac{\mu}{4}\frac{c}{s}\pi \tag{32}$$

$$\int_0^\pi \Gamma_o \sin\theta\cos\theta \, d\theta = \pi C$$

$$\int_0^\pi \Gamma_1 \sin\theta\cos\theta \, d\theta = \frac{\pi}{2}\left\{1 + \frac{c}{s}\frac{\mu}{8}\right\}$$

If these expressions are substituted in equation (28) we obtain

$$\sum_i Q_i \, \delta q_i = -\rho V^2 s^3 \pi \sum_i \delta q_i \, \bar{c} \int_0^1 \left(f_i(\eta)\psi - F_i(\eta)\Psi\right) d\eta \tag{33}$$

where ψ, Ψ, and T are the matrix columns

$$\psi = \left\{ \left(C + \frac{\mu}{4} \frac{c}{s} \right) T , \quad \frac{\mu}{8} \frac{c}{s} T \right\}, \tag{34}$$

$$\Psi = \left\{ \frac{C}{4} \frac{c}{s} T , \left(\frac{1}{8} + \frac{\mu}{64} \frac{c}{s} \right) \frac{c}{s} T \right\} \tag{35}$$

and

$$T = \left\{ T_1, T_3, T_5 \right\} \tag{36}$$

(T_m is defined by equation (4))

If we substitute the value of $\bar{\bar{C}}$ given by (27), equation (33) becomes

$$\sum_i Q_i \ \delta q_i = - \rho V^2 s^3 \pi \sum_i \delta q_i \sum_j q_j \left\{ \mu \bar{\bar{f}}_j - \frac{\mu}{2} \bar{\bar{F}}_j \bar{\bar{H}} \Theta + \bar{\bar{F}}_j \right\} \bar{\bar{W}}^{-1}$$
$$x \int_0^1 (f_i(\eta) \psi - F_i(\eta) \Psi) \ d\eta \tag{37}$$

The generalised aerodynamic forces Q_i, as required for the flutter equations, can be obtained directly from equation (37). It is however of some interest to determine the equivalent constant derivatives, that is derivatives that are constant over the span and that give the same generalised forces. Such derivatives may also be useful in applying the results of this report to wings of slightly different planform (see para. 3.2). For sinusoidal oscillation (i.e. λ purely imaginary), equivalent constant derivatives $(\ell_z)_{ij}$, etc. are defined by analogy with the two-dimensional derivatives, by writing

$$\sum_i Q_i \ . \ \delta q_i = - \rho V^2 s^3 \sum_i \delta q_i \sum_j q_j \left\{ \begin{array}{l} (Z_z)_{ij} + (Z_\alpha)_{ij} + (A_z)_{ij} + (A_\alpha)_{ij} \\ + i \left[(Z_{\dot{z}})_{ij} + (Z_{\dot{\alpha}})_{ij} + (A_{\dot{z}})_{ij} + (A_{\dot{\alpha}})_{ij} \right] \end{array} \right\} \tag{38}$$

where

$$\left.\begin{aligned}
(Z_z)_{ij} &= (\ell_z)_{ij} \int_0^1 f_i(\eta) \,.\, f_j(\eta) \,.\, d\eta \\
(Z_\alpha)_{ij} &= (\ell_\alpha)_{ij} \int_0^1 \frac{c}{s} f_i(\eta)\, F_j(\eta) \,.\, d\eta \\
(A_z)_{ij} &= (-m_z)_{ij} \int_0^1 \frac{c}{s} F_i(\eta) \,.\, f_j(\eta) \,.\, d\eta \\
(A_\alpha)_{ij} &= (-m_\alpha)_{ij} \int_0^1 \left(\frac{c}{s}\right)^2 F_i(\eta) \,.\, F_j(\eta)\, d\eta \\
(Z_{\dot z})_{ij} &= \nu_m\, (\ell_{\dot z})_{ij} \int_0^1 \frac{c}{c_m} f_i(\eta)\, f_j(\eta)\, d\eta \\
(Z_{\dot\alpha})_{ij} &= \nu_m\, (\ell_{\dot\alpha})_{ij} \int_0^1 \frac{c}{c_m} \frac{c}{s} f_i(\eta)\, F_j(\eta)\, d\eta \\
(A_{\dot z})_{ij} &= \nu_m (-m_{\dot z})_{ij} \int_0^1 \frac{c}{c_m} \frac{c}{s}\, F_i(\eta)\, f_j(\eta)\, d\eta \\
(A_{\dot\alpha})_{ij} &= \nu_m (-m_{\dot\alpha})_{ij} \int_0^1 \frac{c}{c_m} \left(\frac{c}{s}\right)^2 F_i(\eta)\, F_j(\eta)\, d\eta
\end{aligned}\right\} \quad (39)$$

and

$$\nu_m = -\frac{i\,\lambda\, c_m}{V} = \frac{\omega\, c_m}{V} \quad (40)$$

is the frequency parameter appropriate to the mean chord c_m. The coefficient $(Z_z)_{ij}$ is obtained from the real part of the term in equation (37) which involves both f_i and f_j; and $(Z_{\dot z})_{ij}$ is obtained from the imaginary part of the same term. The other Z and A coefficients are determined similarly.

It follows from equation (31) that, as ν tends to zero, the two-dimensional lift function C tends to the value

$$1 + \frac{i\nu}{2} \left(\gamma + \log_e \frac{\nu}{4}\right) \quad (41)$$

where γ is Euler's constant, and $\nu = -\frac{i\lambda c}{V} = \nu_m \frac{c}{c_m}$ is the local frequency parameter. It may therefore be deduced from equations (34) to (39) that the derivatives $(\ell_{\dot{\alpha}})_{ij}$ and $(m_{\dot{\alpha}})_{ij}$ obtained by using the method in the above form will be of order $\log_e \nu$ when ν is small. The experimental results quoted by Miss Lehrian[2] suggest that these singularities at $\nu = 0$ should not be present; the error is undoubtedly due to the retention of only a finite number of terms in the series (3), (5) and (6). No practical modification of the method will give the correct values of these two derivatives when the frequency parameter ν is very small at all wing sections. The error which is introduced at flutter frequencies ($\nu_m > 0.2$) due to the very small local frequency parameter of sections near the tip may be corrected, however, by using a value of C based on the mean frequency parameter (instead of allowing C to vary spanwise as is implicit in the above theory). This procedure may be justified as follows.

The function C was introduced into the problem by the assumption that the equivalent doublet distribution was of the form (3) (C occurs in S_o'). It is desirable that any such assumption should satisfy two conditions: (i) the theory must reduce to the two-dimensional theory for very large aspect ratios; (ii) since it is to be expected that the flow over a chordwise strip of the wing, at some appreciable distance from either the centre line or the tip, will not differ greatly from that over a section of a wing of infinite aspect ratio having the same chord, the assumed doublet distribution should be of a form which can reduce approximately to the two-dimensional form for values of η close to 0.5. The form assumed in (3) satisfies these conditions both when C is allowed to vary with the chord c and when C is given a value appropriate to a chord close to the section $\eta = 0.5$, and the latter procedure is therefore justified.*

2.4 Relationship between derivatives for different modes

Suppose the derivatives $(\hat{\ell}_z)_{ij}$ etc. are known for a set of modes** represented by the selected linearly independent functions $\hat{f}_i(\eta)$, $\hat{F}_i(\eta)$. Then the derivatives for a set of given modes

$$f_r(\eta) = \sum_i g_{ri}\, \hat{f}_i(\eta) \qquad (42)$$

$$F_r(\eta) = \sum_i G_{ri}\, \hat{F}_i(\eta) \qquad (43)$$

where the g_{ri}, G_{ri} are constants determined by the above relationships (42), (43) for the known modes $f_r(\eta)$, $F_r(\eta)$, are given by

---

* Tables of C are given in several reports (e.g. references 3 and 4).

** The circumflex does not imply any difference in the physical meaning of the modes f, F, but is used only to denote a particular set of such modes and their associated derivatives.

$$\left.\begin{aligned}
(\ell_z)_{rs} &= \frac{\sum\limits_i \sum\limits_j g_{ri}\, g_{sj}\, (\hat{\ell}_z)_{ij} \int_0^1 \hat{f}_i(\eta)\, .\, \hat{f}_j(\eta)\, d\eta}{\int_0^1 f_r(\eta)\, .\, f_s(\eta)\, .\, d\eta} \\
(\ell_\alpha)_{rs} &= \frac{\sum\limits_i \sum\limits_j g_{ri}\, G_{sj}\, (\ell_\alpha)_{ij} \int_0^1 \frac{c}{s} \hat{f}_i(\eta)\, .\, \hat{F}_j(\eta)\, d\eta}{\int_0^1 \frac{c}{s} f_r(\eta)\, .\, F_s(\eta)\, .\, d\eta}
\end{aligned}\right\} \quad (44)$$

etc.

2.5 Local derivatives

Local derivatives are defined by the relationships:-

$$\text{Lift/unit span} = \rho\, c\, V^2 \sum_j q_j \left\{ (L_z)_j \frac{s}{c} f_j + (L_\alpha)_j F_j \right\} \quad (45)$$

$$\begin{array}{r}\text{Moment/unit span} \\ \text{(Nose up - about} \\ \text{mid chord)}\end{array} = \rho\, c^2 V^2 \sum_j q_j \left\{ (M_z)_j \frac{s}{c} f_j + (M_\alpha)_j F_j \right\} \quad (46)$$

From equations (2), (5) and (10) we see that

$$\left.\begin{aligned}
(L_z)_j &= \frac{\pi}{f_j} \mu \bar{f}_j \bar{W}^{-1} \psi \\
(L_\alpha)_j &= \frac{\pi}{F_j} \frac{s}{c} (\bar{F}_j - \frac{\mu}{2} \bar{F}_j \bar{H} \Theta) \bar{W}^{-1} \psi \\
(M_z)_j &= \frac{\pi}{f_j} \frac{s}{c} \mu \bar{f}_j \bar{W}^{-1} \Psi \\
(M_\alpha)_j &= \frac{\pi}{F_j} \left(\frac{s}{c}\right)^2 (\bar{F}_j - \frac{\mu}{2} \bar{F}_j \bar{H} \Theta) \bar{W}^{-1} \Psi
\end{aligned}\right\} \quad (47)$$

3 Numerical application

3.1 Results

The foregoing analysis was applied to the calculation of derivatives for the delta wing of Fig.1. Values of $\bar{W}^{-1}$ for frequency parameters ν_m of 0, 0.2$\dot{6}$ and 0.8 were obtained from values of $\bar{W}$ calculated by D.E. Lehrian[2]. The values of the reciprocal matrix $\bar{W}^{-1}$ are given in

Table I. The accuracy of the $\bar{W}$ depends upon the fineness of the lattice used in its determination (cf references 1 and 2). No measure of this accuracy can be given and all that can be said is that the finer the lattice the more accurate will be the $\bar{W}$. Miss Lehrian[2] used a 21 x 6 lattice and determined the $\bar{W}$ to an accuracy of seven decimal places and at least five significant figures. This is the accuracy for that particular lattice and not the actual accuracy of $\bar{W}$, which is of course less because of the approximation made in using a finite lattice. The values of $\bar{W}^{-1}$ were determined such that $\bar{W}\bar{W}^{-1} = I$ correct to six decimal places.

Equivalent constant derivatives, as defined by equations (38) and (39), were determined for the modes

$$\hat{f}_i(\eta) = |\eta|^i \qquad \hat{F}_i(\eta) = |\eta|^i \tag{48}$$

where i is given the integral values 0 to 4.

It will be remembered that the general definition of these modes, as given by equation (10), is that f_i represents the normal translation along the mid-chord axis and F_i the change of incidence of fore-and-aft sections. Modes of this form were chosen for simplicity in the aerodynamic problem, and they will not in general be able, individually, to represent realistic structural modes of a sweptback wing. Other modes can however be represented by linear combinations of these functions (see para.2.4), and it should be possible to express practical symmetric modes with reasonable accuracy in terms of the function (48) above. Any chordwise distortion in the modes cannot of course be allowed for, and in this particular application such distortion would have to be neglected.

Values of the derivatives are given in Tables II - IX. These tables also contain, for comparison, values of constant derivatives obtained by using the derivatives of two-dimensional incompressible theory[3] appropriate to the local frequency parameter. These derivatives are defined by the equation

$$(\sigma)_{ij} = \frac{\int_0^1 \sigma\, J_\sigma\, d\eta}{\int_0^1 J_\sigma\, d\eta} \tag{49}$$

where $(\sigma)_{ij}$ is any constant two-dimensional derivative, σ is the corresponding two-dimensional derivative appropriate to the local frequency parameter ν, and J_σ is the integrand in the corresponding expression of equation (39). In the determination of the local derivatives σ the function C (cf equation (31)) was allowed to vary with the local frequency parameter, its value being obtained by the approximate formula suggested by W.P. Jones[4].

The spanwise lift and moment distributions were also determined for the set of modes (48). This was, in effect, the determination of the local three dimensional derivatives defined in section 2.5. The results are given however as values of the lift and moment distribution (Tables X - XIII) rather than values of the derivatives. Figs.2-5 show the distributions for a few typical cases.

3.2 Use of the results

The aerodynamic coefficients for any symmetric modes which have no chordwise distortion can be obtained by the use of equation (37) from the values of $\bar{W}^{-1}$ given in Table I. Their accuracy, however, would be doubtful for modes which have more than two nodes in flexure or two nodes in torsion on each wing. For these it would be necessary to use a finer lattice in determining $\bar{W}$ (cf references 1 and 2) and also to take more collocation points (and hence more terms in the series (3)).

Alternatively, instead of using the matrix $\bar{W}^{-1}$ the aerodynamic coefficients can be determined by means of equations (38) and (39) from the values of the equivalent constant derivatives given in Tables II-IX, provided the modes are the same as those for which the derivatives are tabulated. For other modes having no chordwise distortion which can be written with reasonable accuracy as linear expressions in the modal functions of equations (48) (see equations (42), (43)) the equivalent constant derivatives are obtained by using equations (44), and the aerodynamic coefficients are then determined as before. In the use of these derivatives it must be remembered that they are, by definition, functions only of the modes of distortion and the mean frequency parameter, and do not vary over the span of the wing.

If an approximation to the aerodynamic coefficients for a wing slightly different from that considered in this report is desired, it is probably best to work from the equivalent constant derivatives rather than the values of $\bar{W}^{-1}$. Otherwise there is not a great deal to choose between the two methods.

Acknowledgement

The writer wishes to acknowledge the assistance given by Mr. W.P. Jones and Miss D.E. Lehrian of the National Physical Laboratory.

LIST OF SYMBOLS

A_m $= \frac{s}{c} T_m$

C $= K_1(\lambda c/2V) \big/ \{K_0(\lambda c/2V) + K_1(\lambda c/2V)\}$

$\bar{C}$ see equation (21)

c wing chord

c_m mean wing chord

F_1 mode of change of incidence of fore and aft sections*

$\bar{F}_1$ see equation (23)

f_1 mode of normal translation along the mid-chord axis*

---

* The addition of a circumflex to these symbols is used to denote a particular set of such modes and their associated derivatives.

LIST OF SYMBOLS (Contd.)

$\bar{f}_i$	see equation (24)
H	see equation (20)
I	unit matrix
I_n	unit matrix of order n
$Ke^{\lambda t}$	discontinuity in the velocity potential field
K_o, K_1	modified Bessel functions of the second kind
$(\ell_z)_{ij}$, $(\ell_\alpha)_{ij}$, $(-m_z)_{ij}$, $(-m_\alpha)_{ij}$, $(\ell_{\dot{z}})_{ij}$, $(\ell_{\dot{\alpha}})_{ij}$, $(-m_{\dot{z}})_{ij}$, $(-m_{\dot{\alpha}})_{ij}$	equivalent constant derivatives* - see equation (39)
$(L_z)_j$, $(L_\alpha)_j$, $(M_z)_j$, $(M_\alpha)_j$	- local derivatives* (see equations (45) and (46))
p	air pressure
Q_i	generalized aerodynamic force for i'th degree of freedom
q_i	co-ordinate of i'th degree of freedom
S_n	see equation (3)
s	semi-span
T	see equation (36)
T_m	$= \eta^{m-1}\sqrt{1-\eta^2}$
t	time
V	airspeed
$We^{\lambda t}$	downwash velocity
$\bar{W}$	see equation (22)
W_{nm}	downwash velocity due to a doublet distribution cS_nA_m
x	distance aft of mid chord axis
y	distance spanwise from wing centre line
$ze^{\lambda t}$	downward displacement of any point
$(Z_z)_{ij}$, $(Z_\alpha)_{ij}$, $(A_z)_{ij}$, $(A_\alpha)_{ij}$, $(Z_{\dot{z}})_{ij}$, $(Z_{\dot{\alpha}})_{ij}$, $(A_{\dot{z}})_{ij}$, $(A_{\dot{\alpha}})_{ij}$	see equation (38)

---

* The addition of a circumflex to these symbols is used to denote a particular set of such modes and their associated derivatives.

$\Gamma e^{\lambda t}$		bound vorticity
γ		Euler's constant
η	=	y/s
Θ		see equation (26)
θ	=	$\cos^{-1}(-2x/c)$
λ	=	$i\omega$
μ	=	$\lambda s/V$
ν	=	$\nu_m \, {}^{c}/c_m$
ν_m	=	$\omega c_m/V$
ρ		air density
ϕ		velocity potential
Ψ		see equation (35)
ψ		see equation (34)
ω		circular frequency

REFERENCES

No.	Author	Title, etc.
1	W.P. Jones	The calculation of aerodynamic derivative coefficients for wings of any planform in non-uniform motion. R & M 2470. December, 1946.
2	D.E. Lehrian	Aerodynamic coefficients for an oscillating delta wing. R & M 2841. July, 1951.
3	I.T. Minhinnick	Tables of functions for the evaluation of wing and control surface flutter derivatives for incompressible flow (1950). RAE Report No. Structures 86 - ARC 13,730.
4	W.P. Jones	Aerodynamic forces on wings in non-uniform motion. R & M 2117. August, 1945.

Attached: Tables I to XIII.
Figs. 1 to 5.

Wt.2078.CP170.K3. Printed in Great Britain.

TABLE I

Collocation Points $\eta = 0.2, 0.6, 0.8$

$\theta = \frac{\pi}{2}, \cos^{-1}(-2/3)$

Values of $\bar{W}^{-1}$

(i) $\nu_m = 0$

$\bar{W}^{-1} =$

0.0234735	0.0566361	-0.0606434	1.3681568	-5.2840363	5.0462969
0.0301799	-0.0841849	0.0599972	-0.0904714	5.4788573	-8.2732118
0.0065486	0.0620962	-0.0243123	0.0215825	-1.3397915	3.9486023
0.3291555	-0.8441695	0.7286463	-1.2865950	4.9526629	-4.7599711
0.0146850	0.9999920	-1.4482447	-0.0342545	-4.4331136	7.1749917
0.0198501	-0.2339862	0.7675149	-0.0431344	0.9211493	-3.2734769

(ii) $\nu_m = 0.2\dot{6}$

$\bar{W}^{-1} =$

0.0277237 +0.0055411i	0.0676344 +0.0141885i	-0.0723057 -0.0198116i	1.3678972 -0.0017668i	-5.2874197 -0.0049223i	5.0506522 +0.0098111i
0.0359122 +0.0096079i	-0.1012709 -0.0381417i	0.0718700 +0.0312034i	-0.0916041 -0.0057786i	5.4834242 +0.0290810i	-8.2776384 -0.0310671i
0.0075551 +0.0003698i	0.0740292 +0.0213685i	-0.0280627 -0.0129250i	0.0219779 +0.0011121i	-1.3437240 -0.0138255i	3.9518878 +0.0149722i
0.3911179 +0.1017597i	-1.0172291 -0.3861222i	0.8765592 +0.3525859i	-1.2976105 -0.0590025i	4.9967696 +0.2921388i	-4.8072404 -0.3210517i
0.0174123 -0.0042871i	1.2044997 +0.3419952i	-1.7461317 -0.5180132i	-0.03311249 +0.0108160i	-4.4839188 -0.1867447i	7.2542727 +0.2855609i
0.0221972 +0.0015287i	-0.2866259 -0.0757666i	0.9332836 +0.2447823i	-0.0408064 +0.0019931i	0.9281113 +0.0249720i	-3.3029682 -0.0812751i

(iii) $\nu_m = 0.8$

$\bar{W}^{-1} =$

0.0354173 +0.00461312i	0.0732970 +0.0250602i	-0.0822721 -0.0392212i	1.361251 +0.0064354i	-5.271664 -0.0473006i	5.0378342 +0.0587546i
0.0382815 +0.0207126i	-0.113700 -0.0872222i	0.0777223 +0.0770912i	-0.0836764 -0.0277800i	5.455192 +0.1256427i	-8.2506371 -0.1293859i
0.0116664 -0.00422024i	0.0814245 +0.0447186i	-0.0252463 -0.0363860i	0.0154506 +0.0138403i	-1.3258951 -0.0725773i	3.9334516 +0.0758400i
0.4194435 +0.2141252i	-1.1434979 -0.8844980i	0.9779432 +0.8315600i	-1.2181946 -0.2800632i	4.7080572 +1.2640551i	-4.5417625 -1.3186573i
0.0727322 -0.0685013i	1.3290631 +0.7070830i	-2.0545365 -1.0134346i	-0.1418075 +0.2015937i	-4.1615086 -1.1007569i	7.0136103 +1.3122729i
0.0108417 +0.0107298i	-0.3171515 -0.1502278i	1.1487456 +0.4043349i	-0.0137400 -0.0350107i	0.8395328 +0.2192815i	-3.2707538 -0.3350198i

TABLE II

Values of $(\hat{\ell}_z)_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	0 $0.2\dot{6}$ 0.8	0 -0.01990 -0.3064	0 -0.009691 -0.1418	0 -0.004359 -0.06852	0 -0.002230 -0.04835	0 -0.001270 -0.04393
1	0 $0.2\dot{6}$ 0.8	0 -0.004686 -0.1523	0 -0.003988 -0.07821	0 -0.003677 -0.05597	0 -0.003456 -0.04943	0 -0.003173 -0.04612
2	0 $0.2\dot{6}$ 0.8	0 0.001826 -0.08244	0 -0.001615 -0.05068	0 -0.002809 -0.04411	0 -0.003207 -0.04275	0 -0.003270 -0.04138
3	0 $0.2\dot{6}$ 0.8	0 0.005008 -0.04544	0 -0.0002591 -0.03423	0 -0.002118 -0.03504	0 -0.002841 -0.03662	0 -0.003089 -0.03672
4	0 $0.2\dot{6}$ 0.8	0 0.006672 -0.02385	0 0.0006094 -0.02321	0 -0.001591 -0.02817	0 -0.002504 -0.03165	0 -0.002866 -0.03279

(ii) Two-dimensional

		j		
i	ν_m	0	2	4
0	0 $0.2\dot{6}$ 0.8	0 0.07638 -0.1832	0 0.05312 0.04150	0 0.04341 0.07629
2	0 $0.2\dot{6}$ 0.8	0 0.05312 0.04150	0 0.04341 0.07629	0 0.03797 0.08313
4	0 $0.2\dot{6}$ 0.8	0 0.04341 0.07629	0 0.03797 0.08313	0 0.03440 0.08344

TABLE III

Values of $(\hat{\ell}_\alpha)_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	0 0.26 0.8	1.539 1.497 1.348	1.669 1.624 1.532	1.679 1.634 1.575	1.686 1.640 1.583	1.652 1.606 1.541
1	0 0.26 0.8	1.725 1.676 1.514	1.481 1.447 1.366	1.405 1.378 1.326	1.368 1.343 1.299	1.313 1.291 1.248
2	0 0.26 0.8	1.818 1.766 1.597	1.434 1.404 1.327	1.314 1.293 1.246	1.251 1.235 1.197	1.184 1.169 1.136
3	0 0.26 0.8	1.863 1.810 1.637	1.408 1.381 1.306	1.264 1.247 1.203	1.186 1.173 1.139	1.109 1.099 1.070
4	0 0.26 0.8	1.881 1.828 1.653	1.386 1.361 1.289	1.228 1.213 1.172	1.140 1.130 1.100	1.057 1.050 1.025

(ii) Two-dimensional

		j		
i	ν_m	0	2	4
0	0 0.26 0.8	3.142 2.503 2.070	3.142 2.645 2.258	3.142 2.708 2.352
2	0 0.26 0.8	3.142 2.645 2.258	3.142 2.708 2.352	3.142 2.737 2.401
4	0 0.26 0.8	3.142 2.708 2.352	3.142 2.737 2.401	3.142 2.750 2.425

TABLE IV

Values of $(-\hat{m}_z)_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	0	0	0	0	0	0
	$0.2\dot{6}$	-0.008488	-0.008433	-0.007889	-0.007567	-0.007190
	0.8	-0.04416	-0.03383	-0.02805	-0.02870	-0.03092
1	0	0	0	0	0	0
	$0.2\dot{6}$	-0.007913	-0.005484	-0.004473	-0.004003	-0.003653
	0.8	-0.03992	-0.02616	-0.02149	-0.02024	-0.01959
2	0	0	0	0	0	0
	$0.2\dot{6}$	-0.007516	-0.004447	-0.003326	-0.002812	-0.002465
	0.8	-0.03739	-0.02240	-0.01733	-0.01524	-0.01384
3	0	0	0	0	0	0
	$0.2\dot{6}$	-0.007020	-0.003759	-0.002668	-0.002195	-0.001895
	0.8	-0.03469	-0.01954	-0.01472	-0.01273	-0.01141
4	0	0	0	0	0	0
	$0.2\dot{6}$	-0.006770	-0.003429	-0.002341	-0.001871	-0.001579
	0.8	-0.03355	-0.01777	-0.01283	-0.01075	-0.009410

(ii) Two dimensional

		j		
i	ν_m	0	2	4
0	0	0	0	0
	$0.2\dot{6}$	-0.04365	-0.02532	-0.01843
	0.8	-0.1192	-0.08487	-0.06715
2	0	0	0	0
	$0.2\dot{6}$	-0.02532	-0.01843	-0.01499
	0.8	-0.08487	-0.06715	-0.05673
4	0	0	0	0
	$0.2\dot{6}$	-0.01843	-0.01499	-0.01290
	0.8	-0.06715	-0.05673	-0.04975

TABLE V

Values of $(-\hat{m}_\alpha)_{ij}$

(i) Three-dimensional

i	ν_m	j = 0	1	2	3	4
0	0	-0.3519	-0.3960	-0.3899	-0.3910	-0.3877
	0.26	-0.3478	-0.3890	-0.3798	-0.3787	-0.3742
	0.8	-0.3546	-0.3581	-0.3209	-0.3111	-0.3107
1	0	-0.4119	-0.3671	-0.3512	-0.3450	-0.3341
	0.26	-0.4046	-0.3611	-0.3450	-0.3387	-0.3279
	0.8	-0.3994	-0.3425	-0.3217	-0.3161	-0.3084
2	0	-0.4525	-0.3705	-0.3436	-0.3301	-0.3150
	0.26	-0.4422	-0.3638	-0.3379	-0.3248	-0.3117
	0.8	-0.4220	-0.3424	-0.3173	-0.3060	-0.2935
3	0	-0.4796	-0.3760	-0.3411	-0.3223	-0.3040
	0.26	-0.4687	-0.3701	-0.3368	-0.3187	-0.3010
	0.8	-0.4479	-0.3533	-0.3236	-0.3089	-0.2939
4	0	-0.4973	-0.3797	-0.3393	-0.3168	-0.2961
	0.26	-0.4846	-0.3731	-0.3346	-0.3130	-0.2929
	0.8	-0.4530	-0.3510	-0.3175	-0.2991	-0.2815

(ii) Two-dimensional

i	ν_m	j = 0	2	4
0	0	-0.7854	-0.7854	-0.7854
	0.26	-0.6187	-0.6513	-0.6693
	0.8	-0.5334	-0.5622	-0.5824
2	0	-0.7854	-0.7854	-0.7854
	0.26	-0.6513	-0.6693	-0.6804
	0.8	-0.5622	-0.5824	-0.5975
4	0	-0.7854	-0.7854	-0.7854
	0.26	-0.6693	-0.6804	-0.6843
	0.8	-0.5824	-0.5975	-0.6046

TABLE VI

Values of $(\hat{\ell}_{z})_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	$0.2\dot{6}$	1.487	1.615	1.627	1.634	1.601
	0.8	1.315	1.477	1.515	1.528	1.495
1	$0.2\dot{6}$	1.665	1.440	1.373	1.340	1.288
	0.8	1.475	1.326	1.289	1.268	1.223
2	$0.2\dot{6}$	1.754	1.398	1.289	1.232	1.167
	0.8	1.555	1.291	1.216	1.173	1.117
3	$0.2\dot{6}$	1.798	1.375	1.243	1.170	1.097
	0.8	1.592	1.272	1.176	1.119	1.055
4	$0.2\dot{6}$	1.816	1.355	1.210	1.128	1.048
	0.8	1.607	1.255	1.147	1.081	1.011

(ii) Two-dimensional

		j		
i	ν_m	0	2	4
0	$0.2\dot{6}$	2.459	2.620	2.690
	0.8	1.951	2.173	2.285
2	$0.2\dot{6}$	2.620	2.690	2.722
	0.8	2.173	2.285	2.344
4	$0.2\dot{6}$	2.690	2.722	2.737
	0.8	2.285	2.344	2.376

TABLE VII

Values of $(\hat{\ell}_{\tilde{\alpha}})_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	$0.2\dot{6}$	0.6427	0.6444	0.5662	0.5343	0.5195
	0.8	0.7545	0.7246	0.6339	0.6325	0.6676
1	$0.2\dot{6}$	0.6163	0.5444	0.5399	0.5624	0.5788
	0.8	0.7912	0.6415	0.6123	0.6398	0.6697
2	$0.2\dot{6}$	0.5537	0.5023	0.5280	0.5668	0.5924
	0.8	0.7859	0.6133	0.6014	0.6339	0.6612
3	$0.2\dot{6}$	0.4827	0.4664	0.5152	0.5654	0.5962
	0.8	0.7643	0.5878	0.5882	0.6253	0.6521
4	$0.2\dot{6}$	0.4144	0.4309	0.5008	0.5615	0.5964
	0.8	0.7373	0.5597	0.5724	0.6156	0.6434

(ii) Two-dimensional

		j		
i	ν_m	0	2	4
0	$0.2\dot{6}$	-0.3014	-0.9308	-1.409
	0.8	0.7132	0.3859	0.1500
2	$0.2\dot{6}$	-0.9308	-1.409	-1.765
	0.8	0.3859	0.1500	-0.01901
4	$0.2\dot{6}$	-1.409	-1.765	-2.032
	0.8	0.1500	-0.01901	-0.1458

TABLE VIII

Values of $(-\hat{m}_z)_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	$0.2\dot{6}$ 0.8	-0.3428 -0.3168	-0.3843 -0.3274	-0.3761 -0.3005	-0.3753 -0.2949	-0.3711 -0.2952
1	$0.2\dot{6}$ 0.8	-0.3993 -0.3626	-0.3573 -0.3168	-0.3420 -0.3011	-0.3360 -0.2971	-0.3253 -0.2901
2	$0.2\dot{6}$ 0.8	-0.4374 -0.3924	-0.3611 -0.3249	-0.3359 -0.3044	-0.3232 -0.2950	-0.3086 -0.2836
3	$0.2\dot{6}$ 0.8	-0.4633 -0.4136	-0.3671 -0.3325	-0.3345 -0.3071	-0.3168 -0.2938	-0.2992 -0.2796
4	$0.2\dot{6}$ 0.8	-0.4798 -0.4260	-0.3708 -0.3371	-0.3332 -0.3077	-0.3118 -0.2910	-0.2919 -0.2742

(ii) Two-dimensional

		j		
i	ν_m	0	2	4
0	$0.2\dot{6}$ 0.8	-0.6032 -0.4722	-0.6419 -0.5224	-0.6628 -0.5545
2	$0.2\dot{6}$ 0.8	-0.6419 -0.5224	-0.6628 -0.5545	-0.6753 -0.5748
4	$0.2\dot{6}$ 0.8	-0.6628 -0.5545	-0.6753 -0.5748	-0.6799 -0.5849

TABLE IX

Values of $(-\hat{m}_{\alpha})_{ij}$

(i) Three-dimensional

		j				
i	ν_m	0	1	2	3	4
0	$0.2\dot{6}$ 0.8	0.1421 0.1138	0.2060 0.1513	0.2531 0.1753	0.3059 0.2108	0.3544 0.2501
1	$0.2\dot{6}$ 0.8	0.1732 0.1322	0.1766 0.1345	0.1941 0.1492	0.2192 0.1711	0.2413 0.1914
2	$0.2\dot{6}$ 0.8	0.2033 0.1485	0.1758 0.1337	0.1784 0.1402	0.1908 0.1534	0.2017 0.1648
3	$0.2\dot{6}$ 0.8	0.2291 0.1612	0.1757 0.1336	0.1690 0.1358	0.1757 0.1458	0.1826 0.1546
4	$0.2\dot{6}$ 0.8	0.2601 0.1786	0.1817 0.1346	0.1670 0.1319	0.1693 0.1392	0.1726 0.1457

(ii) Two-dimensional

		j		
i	ν_m	0	2	4
0	$0.2\dot{6}$ 0.8	0.4403 0.1997	0.5647 0.2661	0.6717 0.3207
2	$0.2\dot{6}$ 0.8	0.5647 0.2661	0.6717 0.3207	0.7636 0.3652
4	$0.2\dot{6}$ 0.8	0.6717 0.3207	0.7636 0.3652	0.8363 0.3994

TABLE X

Values of $\frac{1}{\pi}(\hat{L}_z)_j\,\hat{f}_j$

$\nu_m = 0.26$

η \ j	0	1	2	3	4
0	−0.02148 + 0.1638i	−0.002877 + 0.04522i	0.000354 + 0.01765i	0.0006871 + 0.00960i	0.0005538 + 0.00629i
0.1	−0.01922 + 0.1631i	−0.002614 + 0.04605i	0.000325 + 0.01832i	0.0006581 + 0.00997i	0.0005546 + 0.00645i
0.2	−0.01621 + 0.1602i	−0.002562 + 0.04829i	0.000064 + 0.02025i	0.0004913 + 0.01108i	0.0004910 + 0.00697i
0.3	−0.01261 + 0.1552i	−0.002588 + 0.05159i	−0.000334 + 0.02329i	0.0002200 + 0.01295i	0.0003567 + 0.00798i
0.4	−0.00868 + 0.1480i	−0.002548 + 0.05537i	−0.000760 + 0.02719i	−0.0001157 + 0.01563i	0.0001479 + 0.00966i
0.5	−0.00471 + 0.1386i	−0.002317 + 0.05888i	−0.001105 + 0.03158i	−0.0004613 + 0.01908i	−0.0001207 + 0.01219i
0.6	−0.00108 + 0.1268i	−0.001803 + 0.06109i	−0.001257 + 0.03587i	−0.0007450 + 0.02314i	−0.0004104 + 0.01562i
0.7	0.00180 + 0.1122i	−0.000987 + 0.06070i	−0.001128 + 0.03914i	−0.0008834 + 0.02726i	−0.0006508 + 0.01970i
0.8	0.00351 + 0.0937i	0.000037 + 0.05598i	−0.000693 + 0.03985i	−0.0007981 + 0.03023i	−0.0007476 + 0.02345i
0.9	0.00359 + 0.0680i	0.000943 + 0.04380i	−0.000053 + 0.03475i	−0.0004563 + 0.02885i	−0.0005998 + 0.02395i
1.0	0	0	0	0	0

$\nu_m = 0.8$

η \ j	0	1	2	3	4
0	−0.2467 + 0.4273i	−0.03208 + 0.1203i	0.00272 + 0.0474i	0.00504 + 0.02537i	0.002922 + 0.01614i
0.1	−0.2237 + 0.4285i	−0.02949 + 0.1237i	0.00238 + 0.0497i	0.00474 + 0.02660i	0.002951 + 0.01669i
0.2	−0.1944 + 0.4237i	−0.02976 + 0.1308i	−0.00066 + 0.0554i	0.00290 + 0.02986i	0.002286 + 0.01826i
0.3	−0.1603 + 0.4124i	−0.03140 + 0.1406i	−0.00536 + 0.0641i	−0.00010 + 0.03525i	0.000881 + 0.02119i
0.4	−0.1236 + 0.3946i	−0.03269 + 0.1517i	−0.01047 + 0.0752i	−0.00377 + 0.04293i	−0.001208 + 0.02609i
0.5	−0.0864 + 0.3700i	−0.03209 + 0.1620i	−0.01471 + 0.0878i	−0.00745 + 0.05290i	−0.003817 + 0.03346i
0.6	−0.0518 + 0.3384i	−0.02829 + 0.1685i	−0.01678 + 0.1008i	−0.01031 + 0.06476i	−0.006466 + 0.04362i
0.7	−0.0226 + 0.2988i	−0.02068 + 0.1679i	−0.01559 + 0.1101i	−0.01140 + 0.07714i	−0.008380 + 0.05591i
0.8	0.0018 + 0.2488i	−0.00982 + 0.1554i	−0.01072 + 0.1130i	−0.00987 + 0.08655i	−0.008588 + 0.06752i
0.9	0.0081 + 0.1800i	0.00164 + 0.1223i	−0.00311 + 0.0996i	−0.00544 + 0.08367i	−0.006187 + 0.06993i
1.0	0	0	0	0	0

TABLE XI

Values of $\frac{1}{\pi}\frac{c}{s}(\hat{L}_\alpha)_J\,\hat{F}_J$

$\nu_m = 0.26$

η \ J	0	1	2	3	4
0	0.4128 + 0.09692i	0.1140 + 0.01674i	0.04448 + 0.001674i	0.02419 − 0.000735i	0.01583 − 0.000839i
0.1	0.4107 + 0.09068i	0.1160 + 0.01619i	0.04616 + 0.001888i	0.02512 − 0.000572i	0.01624 − 0.000795i
0.2	0.4031 + 0.08126i	0.1216 + 0.01632i	0.05099 + 0.002936i	0.02790 + 0.000104i	0.01756 − 0.000499i
0.3	0.3902 + 0.06920i	0.1298 + 0.01671i	0.05858 + 0.004517i	0.03259 + 0.001180i	0.02007 + 0.000060i
0.4	0.3721 + 0.05535i	0.1392 + 0.01685i	0.06834 + 0.006245i	0.03929 + 0.002505i	0.02428 + 0.000880i
0.5	0.3486 + 0.04073i	0.1479 + 0.01628i	0.07930 + 0.007708i	0.04791 + 0.003879i	0.03060 + 0.001910i
0.6	0.3191 + 0.02664i	0.1534 + 0.01455i	0.08999 + 0.008453i	0.05803 + 0.005029i	0.03918 + 0.003020i
0.7	0.2826 + 0.01442i	0.1524 + 0.01143i	0.09813 + 0.008089i	0.06832 + 0.005640i	0.04937 + 0.003975i
0.8	0.2361 + 0.00535i	0.1405 + 0.00702i	0.09989 + 0.006377i	0.07572 + 0.005374i	0.05872 + 0.004429i
0.9	0.1712 + 0.00039i	0.1100 + 0.00207i	0.08712 + 0.003406i	0.07225 + 0.003917i	0.05995 + 0.003877i
1.0	0	0	0	0	0

$\nu_m = 0.8$

η \ J	0	1	2	3	4
0	0.3681 + 0.3174i	0.1070 + 0.05212i	0.04334 + 0.00482i	0.02333 − 0.00142i	0.01466 − 0.00094i
0.1	0.3669 + 0.2974i	0.1094 + 0.05047i	0.04516 + 0.00557i	0.02435 − 0.00090i	0.01513 − 0.00082i
0.2	0.3609 + 0.2686i	0.1148 + 0.05160i	0.04979 + 0.00926i	0.02704 + 0.00138i	0.01644 + 0.00012i
0.3	0.3501 + 0.2323i	0.1226 + 0.05400i	0.05686 + 0.01484i	0.03144 + 0.00500i	0.01885 + 0.00192i
0.4	0.3349 + 0.1912i	0.1313 + 0.05586i	0.06582 + 0.02095i	0.03767 + 0.00942i	0.02129 + 0.00451i
0.5	0.3149 + 0.1475i	0.1393 + 0.05540i	0.07585 + 0.02616i	0.04574 + 0.01392i	0.02891 + 0.00770i
0.6	0.2895 + 0.1050i	0.1443 + 0.05098i	0.08625 + 0.02889i	0.05536 + 0.01755i	0.03725 + 0.01102i
0.7	0.2571 + 0.0670i	0.1434 + 0.04159i	0.09361 + 0.02775i	0.06546 + 0.01923i	0.04738 + 0.01366i
0.8	0.2145 + 0.0366i	0.1326 + 0.02729i	0.09586 + 0.02192i	0.07320 + 0.01784i	0.05699 + 0.01451i
0.9	0.1543 + 0.0159i	0.1045 + 0.01024i	0.08463 + 0.01171i	0.07077 + 0.01249i	0.05895 + 0.01206i
1.0	0	0	0	0	0

TABLE XII

Values of $\frac{1}{\pi}\frac{c}{s}(\hat{M}_z)_j\,\hat{f}_j$

$\nu_m = 0.26$

η \ j	0	1	2	3	4
0	0.003386 + 0.04487i	0.001190 + 0.009098i	0.0005871 + 0.002088i	0.0003595 + 0.000743i	0.0002431 + 0.000476i
0.1	0.003070 + 0.04088i	0.001076 + 0.008651i	0.0005304 + 0.002144i	0.0003254 + 0.000788i	0.0002208 + 0.000463i
0.2	0.002750 + 0.03668i	0.000978 + 0.008716i	0.0004890 + 0.002574i	0.0003026 + 0.001021i	0.0002065 + 0.000516i
0.3	0.002433 + 0.03230i	0.000892 + 0.009016i	0.0004547 + 0.003209i	0.0002855 + 0.001400i	0.0001958 + 0.000662i
0.4	0.002123 + 0.02779i	0.000807 + 0.009258i	0.0004236 + 0.003902i	0.0002681 + 0.001880i	0.0001844 + 0.000932i
0.5	0.001816 + 0.02317i	0.000716 + 0.009192i	0.0003846 + 0.004470i	0.0002458 + 0.002394i	0.0001697 + 0.001314i
0.6	0.001510 + 0.01850i	0.000610 + 0.008612i	0.0003336 + 0.004754i	0.0002154 + 0.002841i	0.0001496 + 0.001759i
0.7	0.001195 + 0.01385i	0.000485 + 0.007411i	0.0002686 + 0.004609i	0.0001757 + 0.003073i	0.0001236 + 0.002127i
0.8	0.000863 + 0.00930i	0.000343 + 0.005600i	0.0001911 + 0.003921i	0.0001277 + 0.002914i	0.0000920 + 0.002219i
0.9	0.000506 + 0.00501i	0.000191 + 0.003303i	0.0001067 + 0.002619i	0.0000738 + 0.002161i	0.0000553 + 0.001785i
1.0	0	0	0	0	0

$\nu_m = 0.8$

η \ j	0	1	2	3	4
0	0.01880 + 0.1285i	0.003218 + 0.02015i	0.000742 + 0.00145i	0.000546 − 0.000661i	0.0006186 − 0.000223i
0.1	0.01687 + 0.1166i	0.002902 + 0.01960i	0.000680 + 0.00222i	0.000502 − 0.000164i	0.0005650 − 0.000064i
0.2	0.01481 + 0.1036i	0.002973 + 0.02071i	0.000932 + 0.00422i	0.000654 + 0.000961i	0.0006240 + 0.000319i
0.3	0.01274 + 0.1030i	0.003254 + 0.02252i	0.001345 + 0.00687i	0.000903 + 0.002523i	0.0007321 + 0.000981i
0.4	0.01078 + 0.0764i	0.003533 + 0.02408i	0.001747 + 0.00957i	0.001143 + 0.004341i	0.0008408 + 0.001986i
0.5	0.00896 + 0.0628i	0.003626 + 0.02460i	0.001990 + 0.01178i	0.001288 + 0.006181i	0.0008964 + 0.003298i
0.6	0.00730 + 0.0495i	0.003391 + 0.02345i	0.001994 + 0.01309i	0.001279 + 0.007733i	0.0008731 + 0.004741i
0.7	0.00576 + 0.0367i	0.002786 + 0.02035i	0.001667 + 0.01283i	0.001101 + 0.008581i	0.0007627 + 0.005934i
0.8	0.00423 + 0.0246i	0.001891 + 0.01539i	0.001136 + 0.01098i	0.000786 + 0.008230i	0.0005756 + 0.006301i
0.9	0.00257 + 0.0133i	0.000904 + 0.00903i	0.000531 + 0.00733i	0.000409 + 0.006134i	0.0003370 + 0.005113i
1.0	0	0	0	0	0

TABLE XIII

Values of $\frac{1}{\pi}\left(\frac{c}{s}\right)^2 (\hat{M}_\alpha)_j \hat{F}_j$

$\nu_m = 0.26$

η \ j	0	1	2	3	4
0	0.1144 - 0.02053i	0.02324 - 0.005443i	0.00535 - 0.002000i	0.001906 - 0.001032i	0.001216 - 0.0006526i
0.1	0.1041 - 0.01832i	0.02206 - 0.004947i	0.00548 - 0.001857i	0.002017 - 0.000967i	0.001183 - 0.0006114i
0.2	0.0933 - 0.01551i	0.02217 - 0.004489i	0.00655 - 0.001815i	0.002605 - 0.000980i	0.001319 - 0.0006189i
0.3	0.0820 - 0.01244i	0.02287 - 0.004037i	0.00814 - 0.001809i	0.003557 - 0.001024i	0.001688 - 0.0006485i
0.4	0.0704 - 0.00943i	0.02342 - 0.003550i	0.00987 - 0.001780i	0.004760 - 0.001054i	0.002365 - 0.0006740i
0.5	0.0586 - 0.00677i	0.02321 - 0.003001i	0.01128 - 0.001665i	0.006044 - 0.001033i	0.003319 - 0.0006745i
0.6	0.0467 - 0.00464i	0.02170 - 0.002378i	0.01197 - 0.001437i	0.007152 - 0.000936i	0.004430 - 0.0006346i
0.7	0.0350 - 0.00309i	0.01865 - 0.001700i	0.01158 - 0.001099i	0.007723 - 0.000762i	0.005342 - 0.0005475i
0.8	0.0235 - 0.00203i	0.01408 - 0.001026i	0.00984 - 0.000695i	0.007309 - 0.000529i	0.005566 - 0.0004157i
0.9	0.0126 - 0.00123i	0.00829 - 0.000441i	0.00656 - 0.000307i	0.005415 - 0.000274i	0.004471 - 0.0002479i
1.0	0	0	0	0	0

$\nu_m = 0.8$

η \ j	0	1	2	3	4
0	0.1260 - 0.05144i	0.02004 - 0.01115i	0.00161 - 0.003078i	-0.000564 - 0.001387i	-0.000199 - 0.000950i
0.1	0.1130 - 0.04588i	0.01912 - 0.01016i	0.00223 - 0.002909i	-0.000119 - 0.001341i	-0.000180 - 0.000909i
0.2	0.0989 - 0.03859i	0.01969 - 0.00946i	0.00400 - 0.003168i	0.000925 - 0.001587i	0.000331 - 0.001040i
0.3	0.0951 - 0.03049i	0.02090 - 0.00887i	0.00634 - 0.003610i	0.002355 - 0.001978i	0.000957 - 0.001250i
0.4	0.0772 - 0.02255i	0.02191 - 0.00816i	0.00869 - 0.003966i	0.003973 - 0.002305i	0.001858 - 0.001458i
0.5	0.0565 - 0.01553i	0.02202 - 0.00718i	0.01054 - 0.004020i	0.005555 - 0.002469i	0.002988 - 0.001584i
0.6	0.0438 - 0.01002i	0.02071 - 0.00581i	0.01155 - 0.003654i	0.006826 - 0.002361i	0.004190 - 0.001573i
0.7	0.0322 - 0.00621i	0.01775 - 0.00412i	0.01116 - 0.002811i	0.007451 - 0.001965i	0.005146 - 0.001402i
0.8	0.0215 - 0.00387i	0.01327 - 0.00232i	0.00941 - 0.001710i	0.007041 - 0.001348i	0.005387 - 0.001078i
0.9	0.0117 - 0.00236i	0.00770 - 0.00078i	0.00619 - 0.000640i	0.005182 - 0.000657i	0.004325 - 0.000640i
1.0	0	0	0	0	0

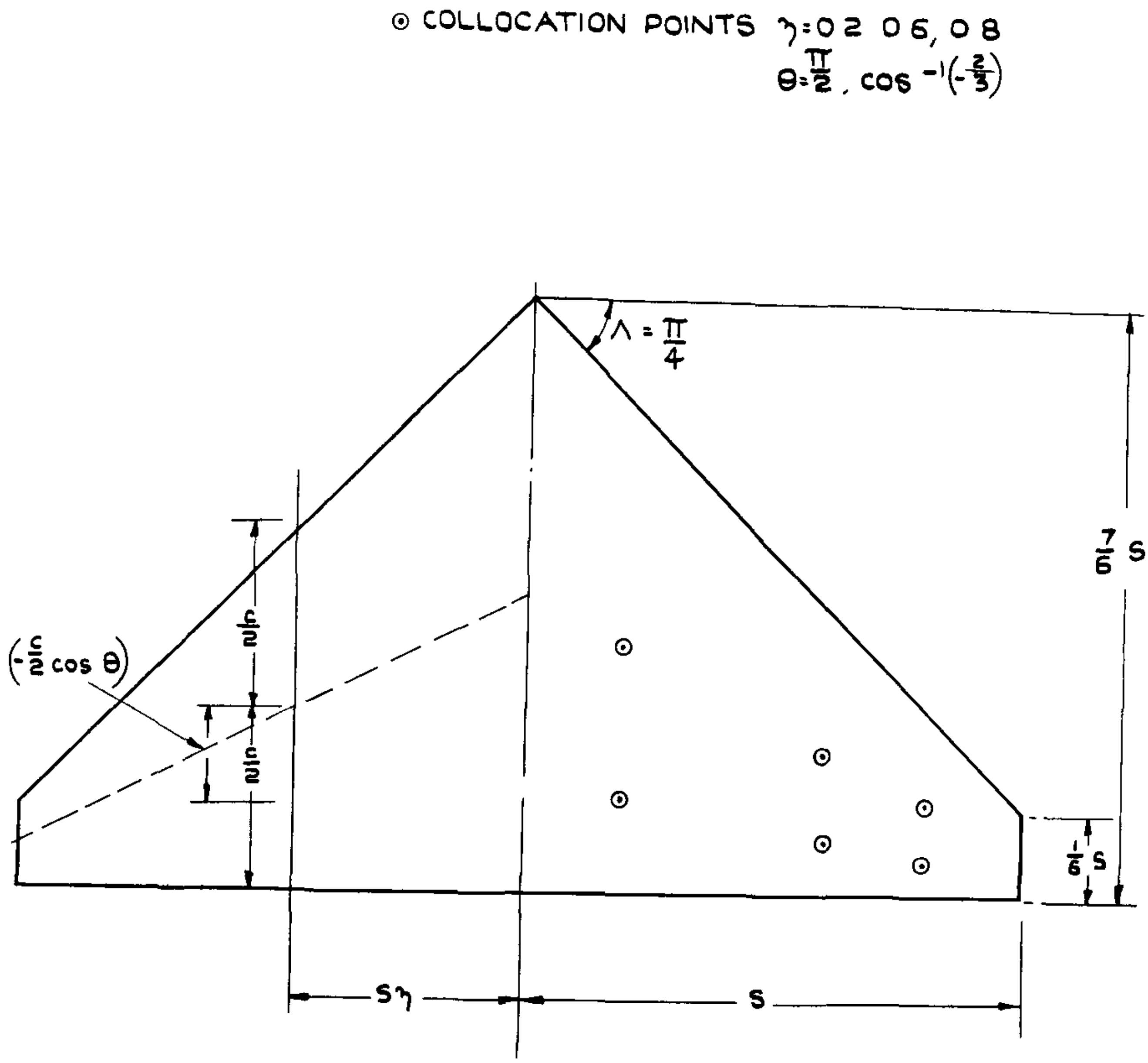

FIG. 1. WING PLAN.

FIG. 2

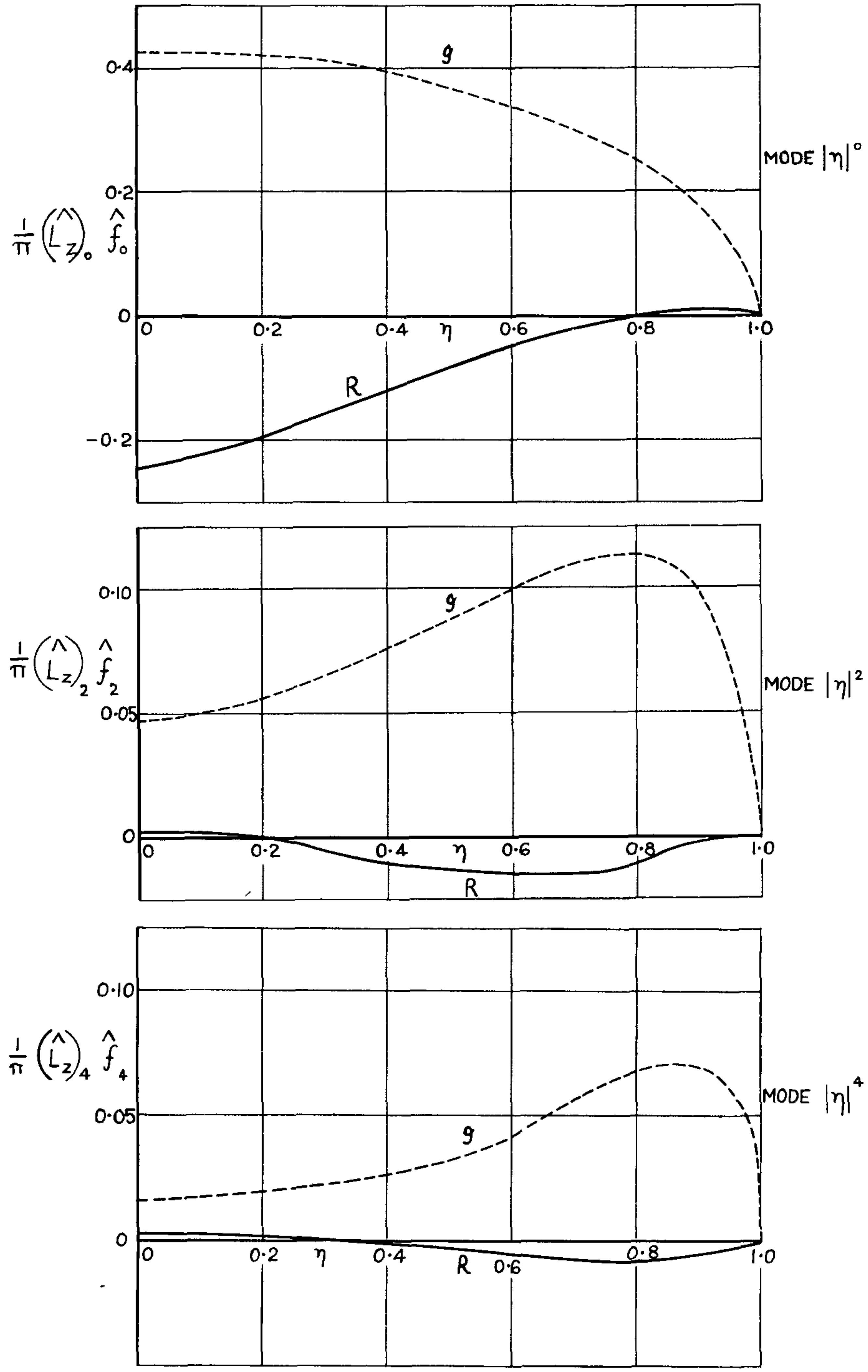

FIG. 2 LIFT DISTRIBUTION FOR FLEXURAL MODES.
$\nu_m = 0 \cdot 8$.

FIG. 3.

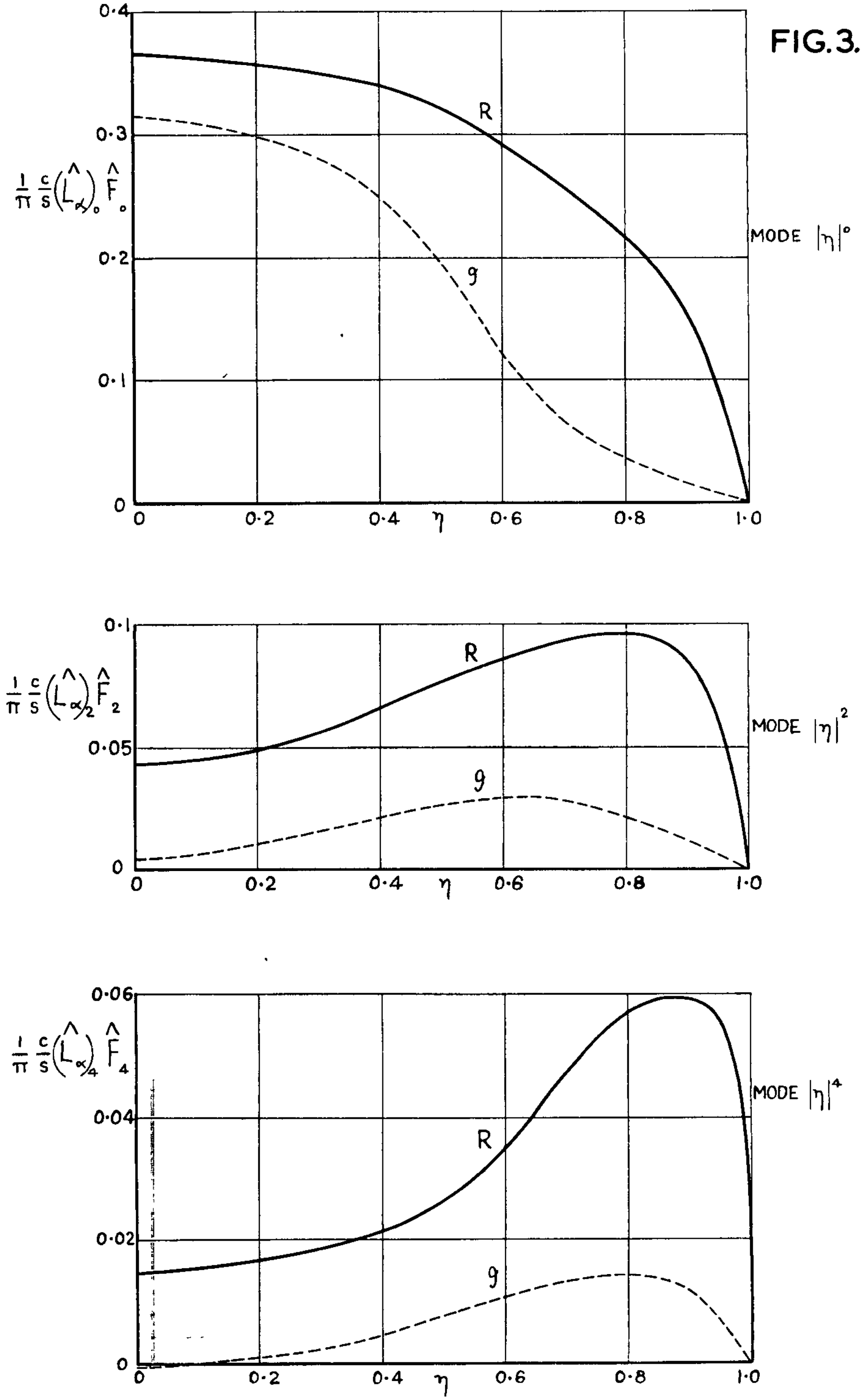

FIG. 3 LIFT DISTRIBUTION FOR TORSIONAL MODES.

$\nu_m = 0{\cdot}8.$

FIG. 4

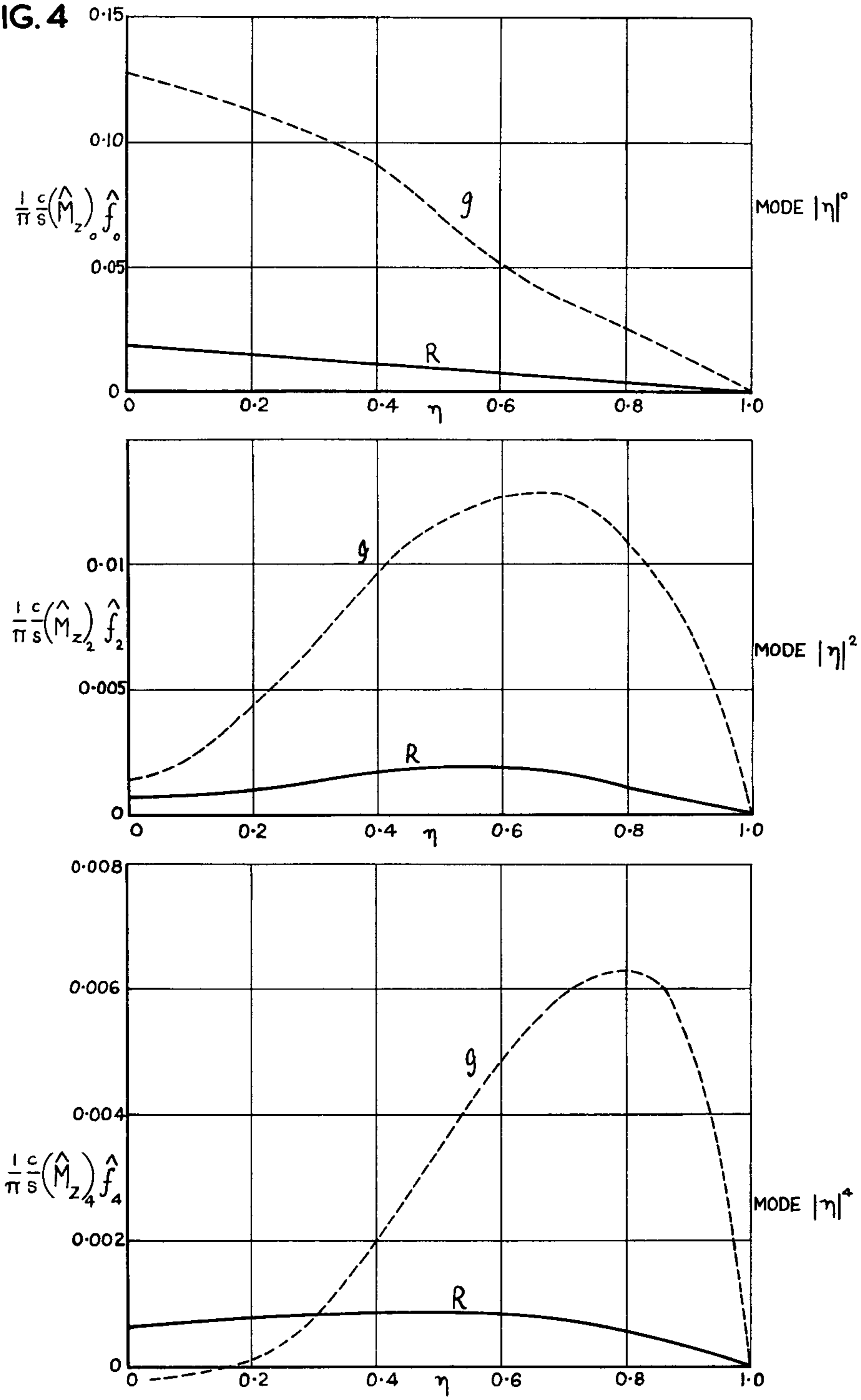

FIG.4 PITCHING MOMENT (ABOUT MID–CHORD) DISTRIBUTION FOR FLEXURAL MODES. $\nu_m = 0{\cdot}8$.

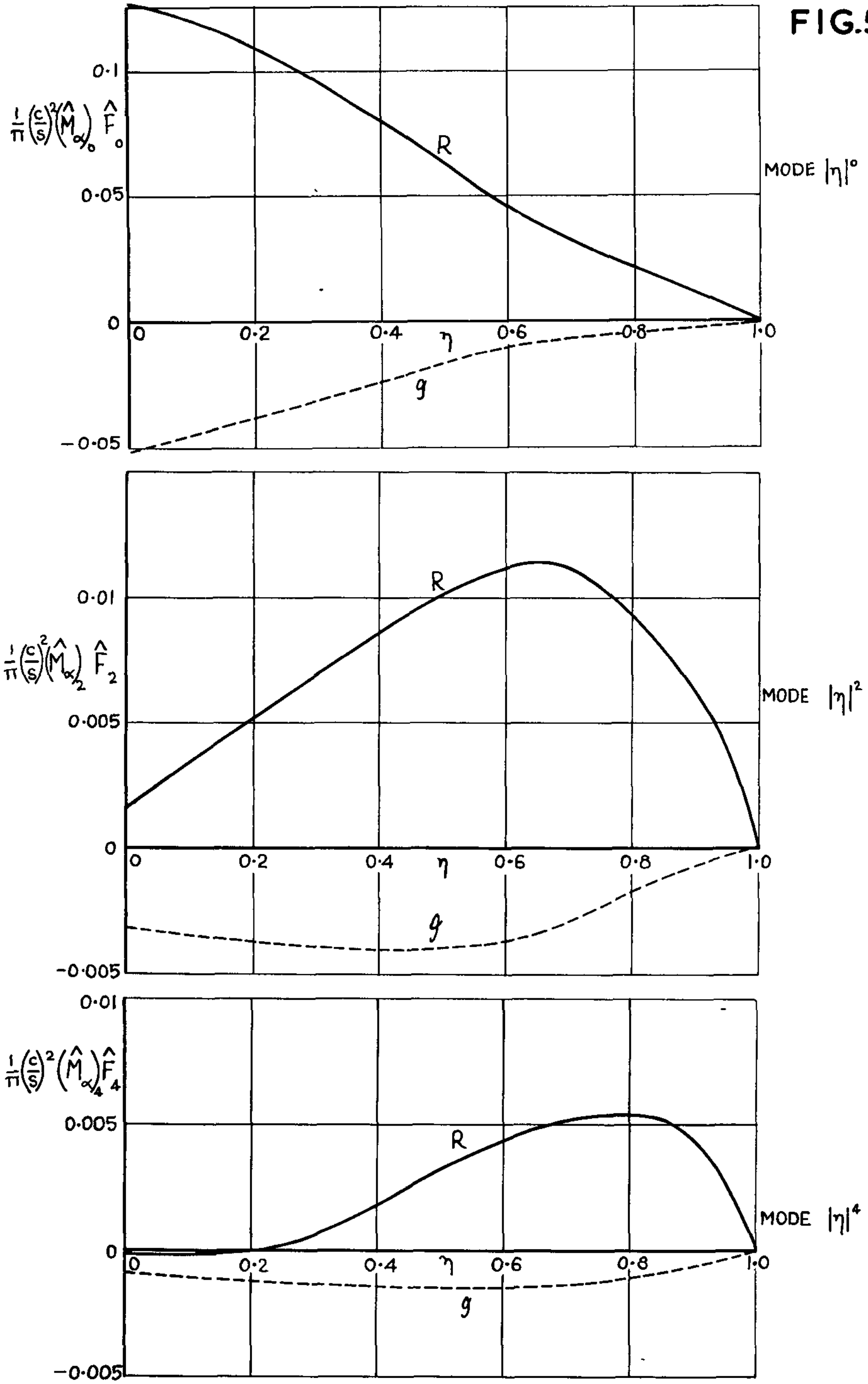

FIG.5 PITCHING MOMENT (ABOUT MID-CHORD) DISTRIBUTION FOR TORSIONAL MODES. $\nu_m = 0{\cdot}8$.

C.P. No. 170
(15,349)
A.R.C. Technical Report

PUBLISHED BY HER MAJESTY'S STATIONERY OFFICE

To be purchased from

York House, Kingsway, LONDON, W C 2. 423 Oxford Street, LONDON, W.1

P O Box 569, LONDON, S E 1

13a Castle Street, EDINBURGH, 2 | 1 St Andrew's Crescent, CARDIFF

39 King Street, MANCHESTER, 2 | Tower Lane, BRISTOL, 1

2 Edmund Street, BIRMINGHAM, 3 | 80 Chichester Street, BELFAST

or from any Bookseller

1954

Price 3s. 0d. net

PRINTED IN GREAT BRITAIN

S.O. Code No. 23-9007-70