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by

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THE ACCELERATION OF WATER DROPS BY AN AIRSTREAM OF CONSTANT RELATIVE VELOCITY

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SUMMARY

A method is suggested for estimating drag coefficients of water drops when the relative airspeed is sufficient to cause distortion of the drops. The results have been used to calculate the acceleration of water drops down vertical ducts designed to give constant airspeed relative to the drops.

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LIST OF SYMBOLS

a	acceleration of drop	ft/sec^2
C_D	drag coefficient based on undistorted diameter	
d	diameter of drop	ft
D	diameter of drop	millimetres
f	drag force on drop	lb wt
F	Froude number	
g	acceleration due to gravity	ft/sec^2
n	acceleration/g	
R	Reynolds number $(\rho_a\ v\ d/\mu)$ ($\approx$ 20.6 vD for standard conditions*)	
s	distance moved by drop	ft
u	speed of drop	ft/sec
v	speed of air relative to drop	ft/sec
W	Weber number $(\rho_a\ v^2\ d/4\gamma)$ ($\approx 3.85\ v^2\ D\ 10^{-4}$ for standard conditions*)	
γ	surface tension	lb wt/ft
μ	viscosity	lb/ft sec
ρ	density	$slug/ft^3$

Suffixes

a	air
c	critical conditions
t	terminal velocity conditions
w	water

* Standard conditions here taken to be 20°C, 750 mm Hg.

1 INTRODUCTION

When making calculations of the acceleration or trajectory of water drops in an airstream it is usual to use drag coefficients based on results for solid spheres. If the airstream velocity is such that the drop is appreciably distorted from the spherical shape drag coefficients based on rigid spheres do not apply and it becomes necessary to use arbitrarily chosen drag coefficients.

In this note a method is suggested for using drag coefficients based on the known terminal velocities of large, distorted drops to derive, for distorted drops of different size, drag coefficients which are thought to be more realistic than those based on solid spheres, for the particular case of constant relative airspeed.

2 THEORETICAL CONSIDERATIONS

The drag of a distorted water drop in an airstream depends on the viscous stresses ($\mu\ dv/dx$), aerodynamic pressures ($\rho_a v^2$) and the shape of the drop. The shape of the drop in turn depends on a balance between aerodynamic ($\rho_a v^2$), surface tension (γ/d) and hydrodynamic pressures ($ng\ \rho_w\ d$) ($n = 1$ for steady vertical motion).

The drag f thus depends on all the above variables and can be written:-

$$f = \phi(\rho_a\ \rho_w\ v\ d\ \gamma\ \mu\ ng) \qquad (1)$$

by a consideration of equality of dimensions this can be expressed as:-

$$f\Big/\tfrac{1}{2}\rho_a v^2 \left(\frac{\pi d^2}{4}\right) = C_D = \phi_1\left(\frac{\rho_w}{\rho_a}\right)\phi_2\left(\frac{\rho_a v^2 d}{4\gamma}\right)\phi_3\left(\frac{\rho_a v d}{\mu}\right)\phi_4\left(\frac{\rho_a v^2}{ng\ \rho_w\ d}\right) \qquad ....(2)$$

$\dfrac{\rho_a v^2 d}{4\gamma}$ the Weber number W, measures the ratio of aerodynamic to surface tension pressures

$\dfrac{\rho_a v d}{\mu}$ the Reynolds number R, measures the ratio of aerodynamic pressure to viscous stress

$\dfrac{\rho_a v^2}{ng\ \rho_w\ d}$ the Froude number F, measures the ratio of aerodynamic to hydrodynamic pressures.

From (2) it is seen that the drag coefficient C_D of a distorted drop cannot be accurately determined by similarity tests but only by a test reproducing all the relevant parameters. A method described in this note for estimating general C_D values from terminal velocity results is one in which ϕ_1 and ϕ_2 are kept constant, a correction to allow for variations in ϕ_3 is assumed and the effect of variations in ϕ_4 is ignored.

For solid spheres or undistorted liquid spheres, ϕ_1, ϕ_2 and ϕ_4 do not apply and $C_D = \phi(R)$. The variation of C_D with R for solid spheres over the range $R = 10^2$ to 10^4 (Ref.1) is shown in Fig.1 at curve A. For convenience drag coefficients of distorted drops will also be plotted against R although this may not, in general, be the most important parameter.

3 DRAG COEFFICIENTS OF FREELY FALLING DROPS

Large drops falling freely in air at their terminal velocity are distorted from a spherical shape and their drag coefficients are larger than for corresponding spheres. The drag coefficients, expressed in terms of undistorted diameter and terminal velocity, can be calculated by equating drag and weight:-

$$\frac{\pi}{8} C_D \rho_a v^2 d^2 = \frac{\pi}{6} g d^3 (\rho_w - \rho_a) \qquad (3)$$

under these conditions the relative airspeed v = terminal velocity u_t

i.e.

$$C_D = \frac{d}{u_t^2} \frac{g}{\rho_a} (\rho_w - \rho_a) \frac{4}{3} \qquad (4)$$

The corresponding value of Reynolds number R is given by:-

$$R = \frac{u_t \; d \; \rho_a}{\mu} \qquad (5)$$

Using the terminal velocities for various drop sizes found in tests at CDEE, Porton drag coefficients have been calculated using equation (4) and are plotted against R in Fig.1 as curve B. The results are summarised in Table 1. Drops smaller than about 1 mm diameter falling at their terminal velocity are apparently little distorted from a spherical shape as the C_D values agree quite well with the solid sphere values whilst drops approaching 7.24 mm diameter falling at their terminal velocity have drag coefficients differing greatly from the sphere values.

4 DERIVED DRAG COEFFICIENTS

Using an average pressure distribution applicable to spheres over the range of Reynolds number 10^3 to 10^5 Savic has shown[2] that for small distortions the amplitude of distortion of a liquid drop in an airstream is proportional to the Weber number W.

This result is used to support a suggested method for deriving general C_D values for distorted drops from terminal velocity values. Let a drop of initial undistorted diameter d, have a known drag coefficient C_{D_0} when falling at its terminal velocity. The Reynolds number is then R_1 and the relative airspeed v_1. It is assumed that a second drop of diameter d_2 will have the same distorted shape if the Weber number is the same. The relative airspeed required to ensure this is given by

$$v_1^2 \, d_1 = v_2^2 \, d_2 \qquad (6)$$

From (5) and (6) the Reynolds number R_2 of the second drop is given by:-

$$R_2 = R_1 \times (d_2/d_1)^{\frac{1}{2}} \qquad (7)$$

As the two drops are assumed to have the same shape the drag coefficients are assumed to be the same apart from a correction to allow for the change in Reynolds number from R_1 to R_2. In the change from R_1 to R_2 the drag coefficient of a sphere changes from C_{D_1} to C_{D_2} which can be found from curve A of Fig.1. It is assumed that the drag coefficient of the drop changes in the same ratio so that the drag coefficient C_D of the drop of diameter d_2 at Reynolds number R_2 is given by:-

$$C_D = C_{D_0} \times \frac{C_{D2}}{C_{D1}} \qquad (8)$$

By means of equations (7) and (8) a curve of C_D against R for a drop of a given size can be derived from the terminal velocity results.

To determine the point where the derived drag curves should depart from the spheres drag curve the Weber number relation is again used. The drag curve for freely falling drops departs from the spheres curve at a Reynolds number of approximately 272 which corresponds to a 1 mm diameter drop falling at its terminal velocity of 13.2 ft/sec. It is assumed that the drag curve for a drop of diameter d_2 will depart from the spheres drag curve when the Webers number is the same as that for the 1 mm diameter drop. From (7) this occurs at a Reynolds number given by:-

$$R_2 = 272 \times (d_2/d_1)^{\frac{1}{2}} = 272 \times D_2^{\frac{1}{2}} \qquad (9)$$

The derived drag coefficient curve for a 2 mm diameter drop using the above method is shown in Fig.1 at curve C which also shows the steps in deriving a point on the curve from the terminal velocity data for a drop of 5.42 mm diameter. Fig.2 shows the derived curves for drops of $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2 and 3 mm diameter. The minimum airspeed relative to a drop under steady conditions is its terminal velocity, unless the drop is in an airflow directed vertically downwards. For this reason the derived drag curves are shown dotted for values of R less than that corresponding to the drop terminal velocity.

If the relative airspeed over the drop is steadily increased the distortion increases until a critical speed is reached when the distortion is excessive and the drop breaks-up. This critical speed defines a maximum value of Reynolds number for the drop beyond which the derived drag curve must not extend. Tests at CDEE, Porton[3] on the break-up of drops up to a diameter of 4.69 mm in a steady airstream establish the following relation between drop size and critical relative airspeed:-

$$v_c^2 \, d = 21.6 \qquad (10)$$

This implies that the critical Weber number is constant.

The 7.25 mm diameter drop in the terminal velocity results is approximately the largest drop which is stable when falling at its terminal velocity and this is only slightly less than the critical speed calculated from (10). Thus the Reynolds number for any given drop size derived from the 7.25 mm drop results on the basis of constant Weber number will also be slightly less than the corresponding critical Reynolds number.

5 APPLICATION OF RESULTS

Strictly, the derived results can only be applicable in cases where the airspeed relative to the drop is constant or, possibly, varying only slowly with time. Such a case is that of the acceleration of a drop down a vertical converging air duct shaped in such a way as to maintain constant relative airspeed. A duct of this type could be used for certain raindrop impingement tests.

The equation of motion of the drop is given by:

$$\frac{\pi}{6}(\rho_w - \rho_a)\, d^3\, a \;=\; C_D \frac{\pi}{8} \rho_a v^2 d^2 + \frac{\pi}{6}(\rho_w - \rho_a)\, g\, d^3 \qquad (11)$$

where v is the constant relative airspeed and a is the constant acceleration of the drop.

Thus

$$a \doteqdot \left(C_D \frac{3}{4} \frac{\rho_a}{\rho_w} \frac{v^2}{d}\right) + g \qquad (12)$$

the speed of the drop is then given by

$$u^2 - u_o^2 = 2as \qquad (13)$$

where s is the distance moved by the drop down the duct,

or

$$u = \left\{\left[\left(C_D \frac{3}{2} \frac{\rho_a}{\rho_w} \frac{v^2}{d}\right) + 2g\right] s + u_o^2\right\}^{\frac{1}{2}} \qquad (14)$$

To obtain the highest drop speeds the largest values of v and C_D are required consistent with not unduly distorting the drop. It has been assumed that the distortion depends primarily on the Weber number W which has a maximum value of 2.54 at the point of break-up of the drop. A value of W = 1.5 has been chosen to determine the distortion in this case which corresponds to a drop of 4.5 mm diameter falling at its terminal velocity of 29.5 ft/sec. The theoretical shape is intermediate to the two shapes calculated by Savio[2] for W = 1 and W = 2 and shown in Fig.3.

The values of v and C_D corresponding to W = 1.5 for various drop sizes are given below

Dia mm	v ft/sec	C_D
1	63	0.67
2	44	0.65
3	36	0.63

Using these values in equation (14) the speed of 1, 2 and 3 mm drops down vertical ducts has been calculated for the case $u_o = 0$ and the results plotted in Fig.4. For comparison the corresponding speed of a freely falling body is also shown. It can be seen that the smaller drops can be accelerated most rapidly and that very long ducts are required.

LIST OF REFERENCES

No.	Author	Title, etc.
1	Goldstein, S.	Modern developments in fluid dynamics. Oxford University Press.
2	Savic, P.	Circulation and distortion of liquid drops falling through a viscous medium. N.R.C. Report MT-22, dated 31/7/53.
3	Lane, W.R., Edwards, J.	The break-up of drops in a steady stream of air. Internal M.O.A. Report.

TABLE 1

Terminal velocity data

Standard atmospheric conditions of 20°C and 750 mm Hg.

Diameter mm	u_t ft/sec	R	log R	C_D	W
0.69	9.11	129.5	2.112	0.973	0.022
0.77	10.45	165.8	2.220	0.829	0.032
0.79	10.32	167.9	2.225	0.871	0.032
2.29	22.84	1078	3.033	0.513	0.459
3.38	27.10	1886	3.276	0.539	0.955
4.07	28.72	2410	3.382	0.578	1.291
4.42	29.48	2685	3.429	0.597	1.476
4.93	29.58	3009	3.478	0.661	1.658
5.42	30.10	3360	3.526	0.701	1.886
5.95	30.13	3693	3.567	0.770	2.075
6.24	29.80	3833	3.584	0.823	2.130
6.37	29.96	3933	3.595	0.831	2.200
6.67	30.05	4130	3.616	0.865	2.316
7.25	30.04	4485	3.652	0.942	2.514

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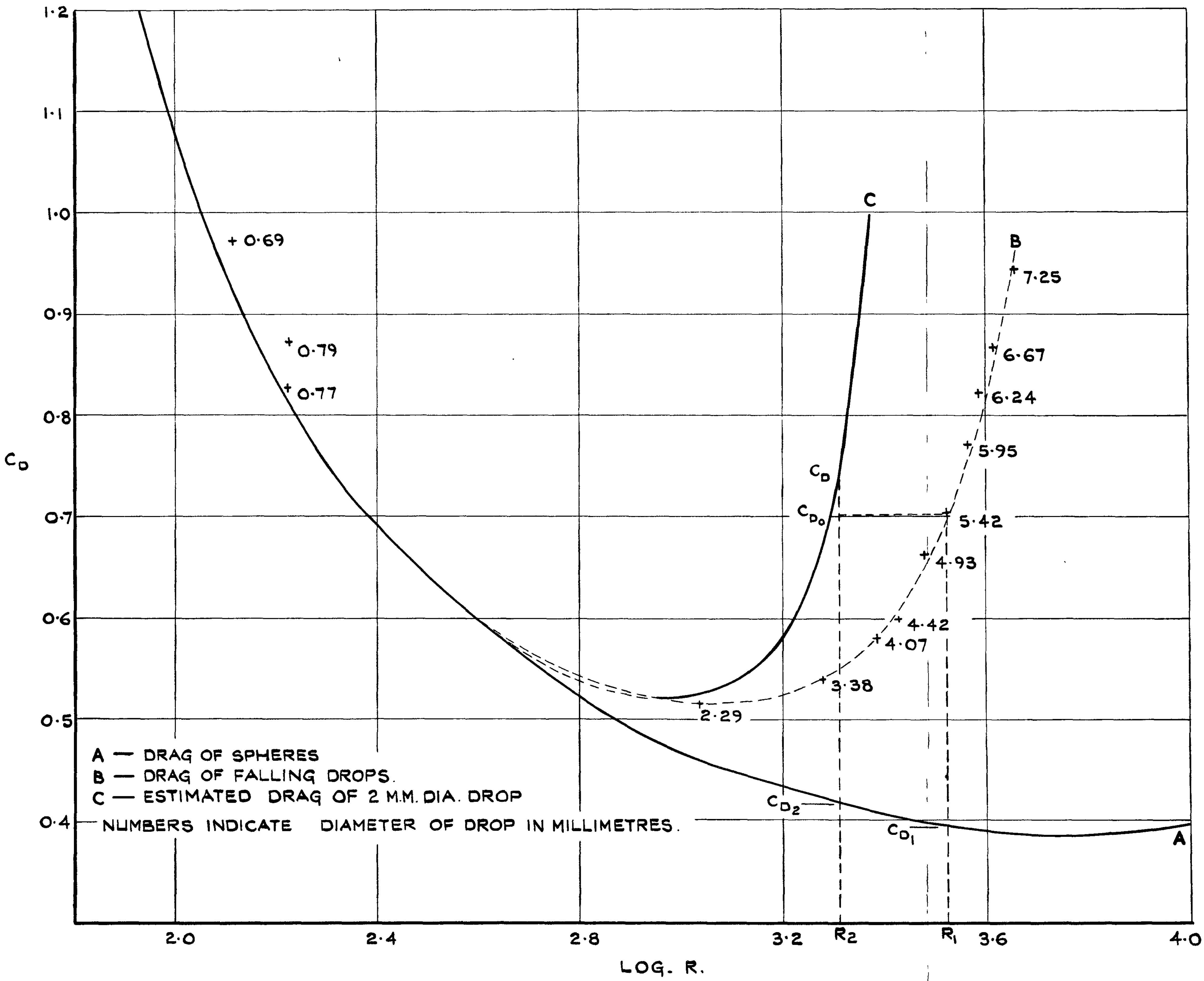

FIG. 1. DRAG COEFFICIENTS FOR SPHERES AND DISTORTED WATER DROPS.

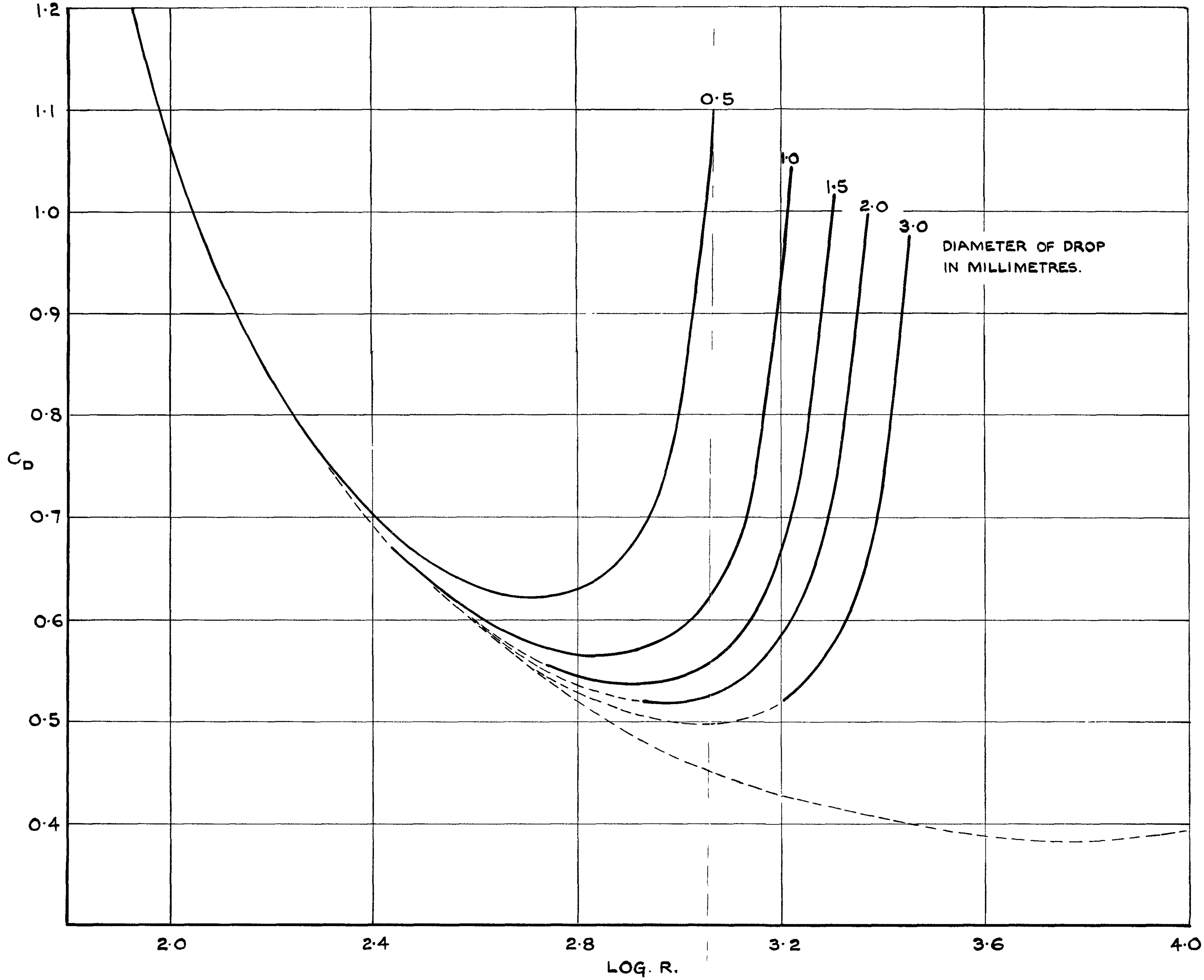

FIG. 2. ESTIMATED DRAG COEFFICIENTS FOR DISTORTED WATER DROPS OF VARIOUS SIZES.

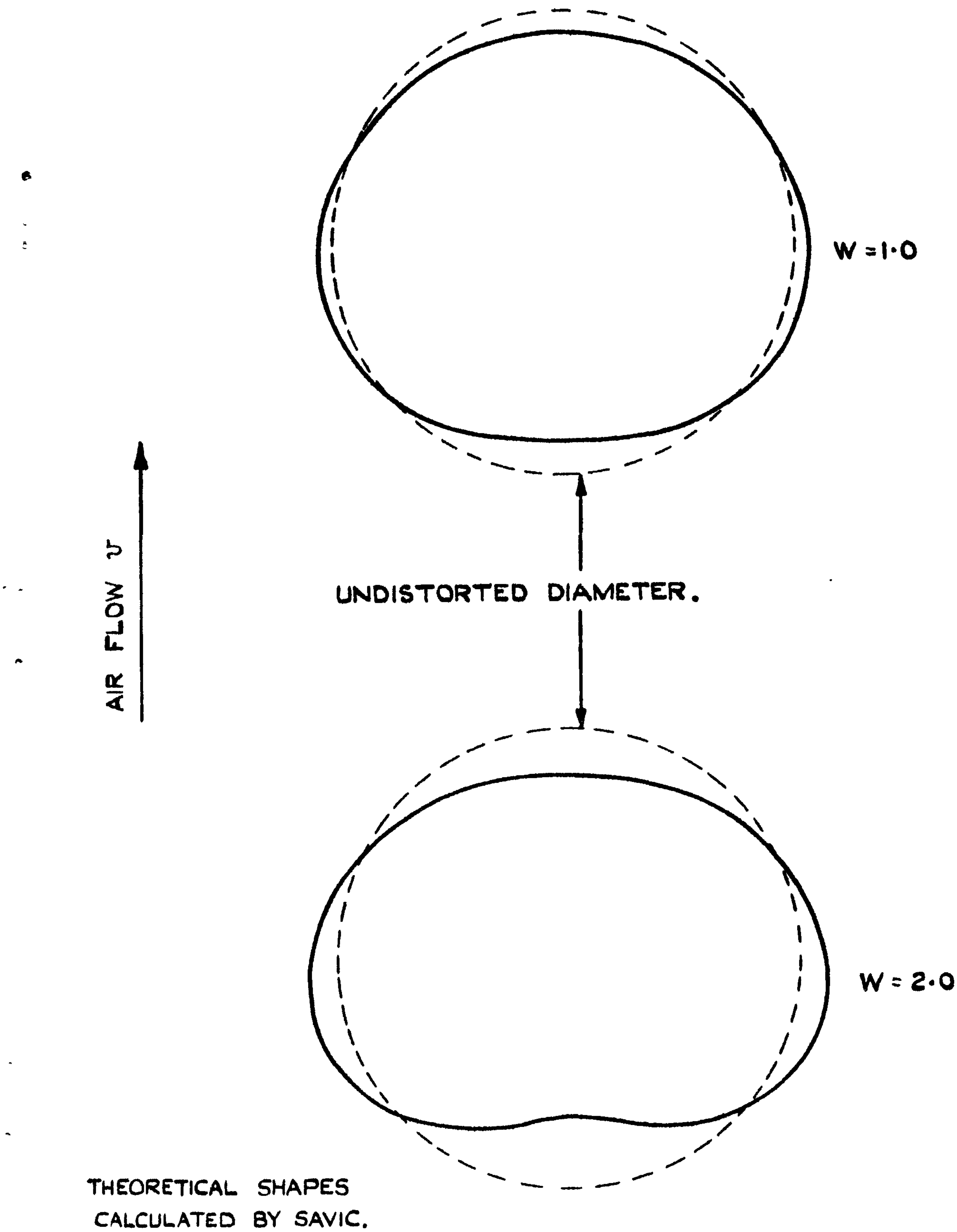

FIG. 3. THEORETICAL DISTORTION OF DROPS IN AN AIRSTREAM.

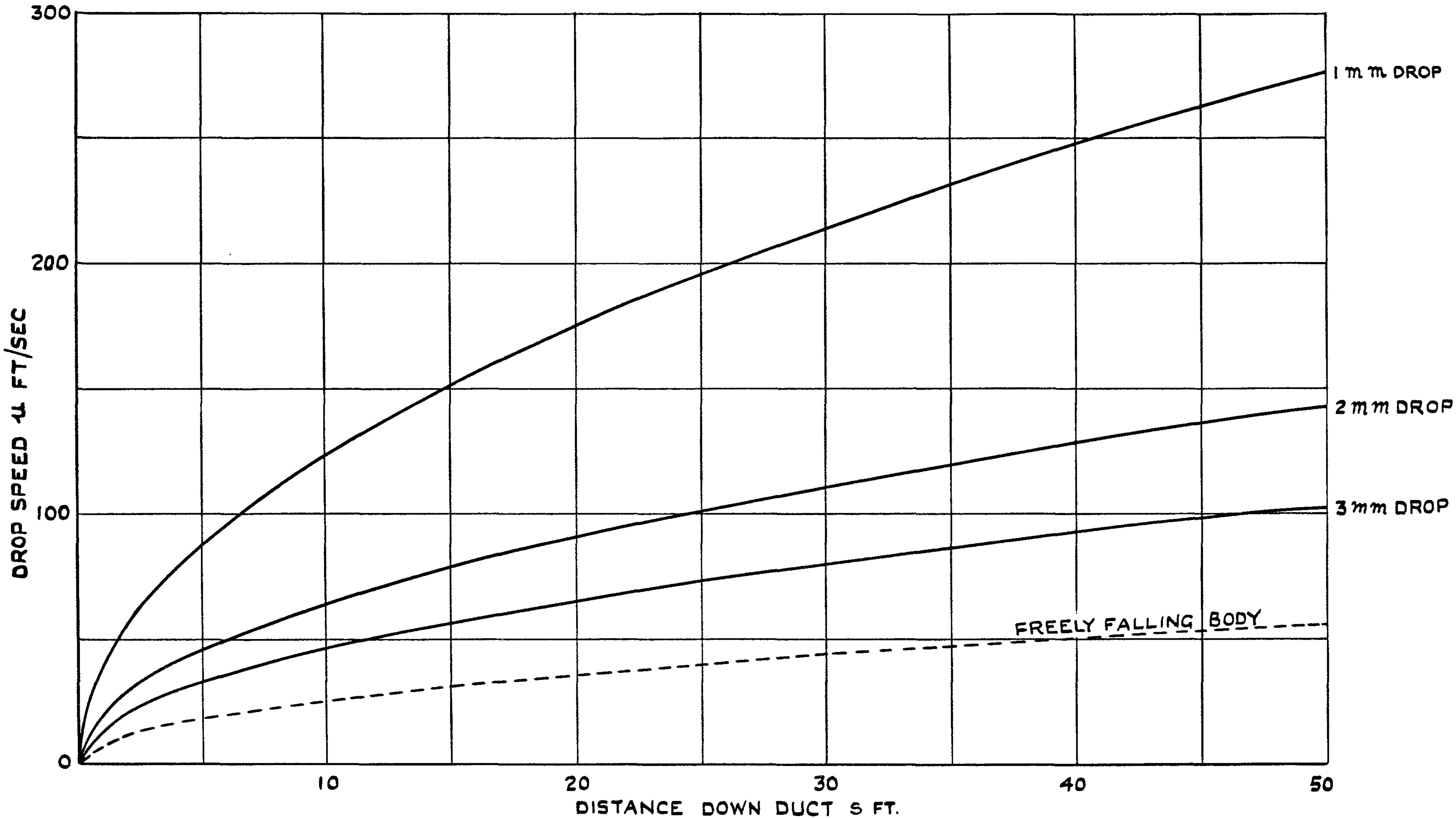

FIG. 4. ACCELERATION OF DROPS DOWN A SPECIALLY SHAPED DUCT.