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The Calculation of Optimum Incidences for Aerofoils

By

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The Calculation of Optimum Incidences for Aerofoils

- By -
A. D. S. Carter

May, 1961

SUMMARY

It was considered that existing methods for calculating the optimum incidence of aerofoils in cascade are so empirical that they could not be extended to new profiles without considerably more supporting test work. A more fundamental approach involving less empiricism was therefore sought. By associating the optimum incidence with a fixed position of the front stagnation point on the leading edge a method is suggested which enables this incidence to be calculated from a knowledge of the reversed flow (i.e., the flow through a turbine cascade if the incidence for a compressor cascade is required and vice versa).

The method has been applied to a wide range of compressor cascades and found to give satisfactory results, except perhaps at very low cambers. The method applies essentially to profiles formed by using the C50 camber line for which extensive test information and experience is available. It also agrees with all the more limited test data available on the P40 camber line. It is therefore assumed that it would apply to other profiles likely to be used in axial flow compressors. Since the method involves only one empirical constant it is thought that it may be used with much greater confidence for cascades outside the range of present experience.

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1./

Replaces A.R.C.22,855.

1. Introduction

The possibility of using different blade profiles for various compressor duties has raised once again the question of estimating design values of incidence and deviation. The latter would not in fact seem to offer much difficulty. It can be estimated from the potential flow past the cascade of blades with sufficient practical accuracy. Existing "deviation rules" would appear to cover a wide enough range of profile stagger and pitching if reasonable interpolation is allowed. The design incidence is much more difficult to estimate, partly because it is a loosely defined quantity anyway, and partly because there is no theoretical basis for comparison of test data. "Incidence rules" have therefore been mainly empirical.

It is often considered that there is no great need for refinement in estimating the working incidence because small errors can be accommodated by the blade. By comparison any error in deviation has, of course, a very marked effect on performance. Deviation has consequently received considerable attention. However this view does not tally with the practical experience, gained during compressor development, that twisting or skewing blades by a few degrees can modify stall and surge of a compressor to a very marked extent. This experience suggests that incidence is a most important factor, and the extrapolation of existing rules to meet some of the profiles suggested in Refs.1 and 2, for example, would be quite unjustified.

While it is known that private firms etc. in this country have their own incidence data, the only published information, so far as the author is aware, is given in Refs.3, 4 and 5, and this is confined solely to blades on circular arc camber lines. The present paper first attempts to put the design incidence for these blades on to a firmer foundation. The method is then applied to the limited test results on other profiles.

2. Basis of Analysis

2.1 Criticism of previous work

In Ref.3 a probably positive step forward was taken in relating the performance of a cascade of blades to the incidence corresponding to maximum lift/drag ratio, which was designated optimum incidence (i_{opt}). The lack of precision which is associated with stalling, and which therefore applies to the nominal incidence (i^*) was thereby removed. Reviewing the general argument of Section 2.1 of Ref.3 it would appear that the second step associating the optimum incidence with a fixed position of the front stagnation on the aerofoil profile was also sound. Had the problem then been solved by potential flow calculations, as was done for deviation, little objection could be taken to the analysis. However the method of by-passing this step would seem to be open to several objections.

The method consists of replacing each of the aerofoils, except the one under consideration, by a concentrated vortex. It is not difficult to see that this is a gross approximation for the low pitch/chord ratios at which practical designs are carried out. Secondly the exact Betz function for v_n^*, the velocity perpendicular to the chord line at the leading edge, is replaced by an entirely empirical function. It must be emphasised that even though the exact Betz function was used in the first step in the calculation, the <u>whole</u> of the derived function is empirical. It would appear therefore that there is no reason whatsoever for assuming that it applies to any profile other than that for which the original analysis was intended. If the empirical function was more solidly dependent on the theoretical function (such as a constant proportion of it, or a constant stagger or pitch adjustment) greater reliance may have been

placed/

placed on the method. In the existing circumstances however the author feels it must be rejected for extended work, unless extensive test results are available for a repeated analysis. This is not the case.

2.2 Revised analysis

The revised analysis presented in this paper is again based on the assumption that the optimum incidence is associated with a fixed position of the front stagnation point. If we consider in the first instance the flow of an ideal fluid past a cascade of blades whose profiles have sharp leading edges, then the front stagnation points must be located at those leading edges if infinite velocities are to be avoided. Our problem would then be to calculate this incidence without resort to lengthy potential flow calculations. This is a simple problem in the case of profiles which are symmetrical about the mid-chord position. We need only reverse the direction of the flow, as illustrated in Fig.1, to see that the flow pattern at optimum incidence must be the same for an identical profile in compressor and turbine configurations of the same numerical stagger, at the same pitching. The optimum incidence in this case is then defined by the Kutta-Joukowski condition for reversed flow. Thus for these profiles we can write:

$$\left\{\begin{matrix}\text{Optimum Incidence as} \\ \text{Compressor Cascade}\end{matrix}\right\} = - \left\{\begin{matrix}\text{Deviation as} \\ \text{Turbine Cascade}\end{matrix}\right\} \qquad \ldots (1)$$

and

$$\left\{\begin{matrix}\text{Optimum Incidence as} \\ \text{Turbine Cascade}\end{matrix}\right\} = - \left\{\begin{matrix}\text{Deviation as} \\ \text{Compressor Cascade}\end{matrix}\right\} \qquad \ldots (2)$$

Equations (1) and (2) can be combined and written symbolically as

$$i_{opt} = - \delta\,[(-\zeta)]\,. \qquad \ldots (3)$$

Equation (3) only applies to symmetrical profiles with sharp leading and trailing edges. It is known that the camber line has considerable influence on the deviation, but so long as we are concerned only with parabolic arc camber lines - and most practical camber lines can be replaced by a parabolic arc with sufficient accuracy - the condition of reversed flow can be introduced into equation (3) by writing

$$i_{opt} = - \delta\left[(-\zeta),\ \left(1 - \frac{a}{c}\right)\right]. \qquad \ldots (4)$$

Thus equation (4) may be expected to give the optimum incidence for any profile having sharp leading and trailing edges.

Rounded leading edges together with different thickness distributions introduce a new factor. One might expect that it would be possible, and desirable, to operate profiles having rounded leading edges at incidences above that at which the stagnation point occurs at the leading edge. If we denote the additional incidence that can be imposed above that giving leading edge stagnation by i_a, the optimum incidence will be given by,

$$i_{opt} = i_a - \delta\left[(-\zeta),\ \left(1 - \frac{a}{c}\right)\right]. \qquad \ldots (5)$$

The additive incidence i_a will be a function of the pressure distribution around the blade. Since this is known to be substantially the same for any

profile/

profile over limited practical ranges of camber stagger and pitching, it would seem reasonable to expect that i_a will be constant for any profile used within that range.

We may speculate at least on the qualitative variation of i_a with profile, e.g., with thickness distribution. It is easily seen, as in Fig.1, that a section which has its peak upper surface velocity near the leading edge as a compressor will have it near the trailing edge in reversed flow as a turbine. Thus although the potential flow can be reversed, the real flow cannot since the boundary layer conditions must differ. As pointed out in Ref.1, incidence effects are confined very largely to the leading edge region for blades in cascade. If therefore the peak velocity on the upper surface is already near the leading edge it may be anticipated any substantial increase above that giving leading edge stagnation will result in separation. On the other hand if the peak velocity on the upper surface is well back a much larger increase of incidence above that giving leading edge stagnation may be anticipated. Speaking in general terms therefore one may expect higher values of i_a for a high speed section (which usually has its peak surface velocity towards the rear) than for a low speed section. Reference must necessarily be made to test results, before any qualitative variation of i_a can be given.

In concluding this section it must be noted that the suggested method of estimating the optimum incidence necessitates only one additional empirical factor over that necessary to estimate the deviation in reversed flow. Methods of accomplishing the latter are well established. It is probable that the empirical factor will remain constant for a given profile, and its qualitative variation with profile has been established.

3. Comparison with Test Results

3.1 Circular arc camber lines

The performance of the C4 profile on circular arc camber lines appears to provide the best means for testing the above hypothesis. Considerable test data is available at both negative and positive staggers, even though it is an unrepresentative turbine profile. For the last reason, and also because we are at present mainly interested in compressor cascades the following analysis is confined to negative staggers.

It first becomes necessary to estimate the deviation in reversed flow, i.e., for a circular arc cambered blade at positive stagger. The most general rule gives

$$\delta = m\,\theta\,\frac{s}{c} \qquad \ldots (6)$$

where m has been plotted in Fig.2 from Ref.6. However the variation of m over the range of staggers is so small that it was decided to take it as a constant as in Reeman's rule (Ref.7). This gives

$$\delta = 0.19\,\theta\,\frac{s}{c}\,. \qquad \ldots (7)$$

There is considerable theoretical and test evidence to substantiate this rule (see Refs.6 and 7) and hence great reliance can be placed upon it.

Since the detailed test data used in Ref.3 remains unpublished it was decided to use the values of the optimum incidence calculated from Ref.3 as a comparison. These are widely used and quoted so that this comparison has some practical significance.

As/

As a result of examination it was found that a mean value of $i_a = 6.5°$ could be used for the range of camber angles from 10° to 50°. Thus equation (5) becomes

$$i_{opt} = i_a - \delta\left[(-\zeta), \left(1 - \frac{a}{c}\right)\right]$$

$$= 6.5° - 0.19\,\theta\frac{s}{c}\,. \qquad \ldots (8)$$

The values of the optimum incidence calculated by means of equation (8) have been plotted against the camber angle (θ) for pitch/chord ratios of 0.5, 1.0 and 1.5 in Figs.3, 4 and 5 respectively. Also plotted on these figures are the values given in Refs.4 and 5 which have been calculated from Ref.3. It is thought that the agreement is reasonably good for this kind of work over the camber range 20° to 50°, and acceptable for most practical purposes for the range 10° to 50°. The only serious discrepancy occurs with a combination of low pitching, low camber and high outlet angle. The optimum incidence calculated from equation (8) errs on the safe side in this case.

The lack of agreement at low cambers is not altogether surprising. The deviation for a zero cambered blade is not zero as indicated by the deviation rule of equation (7), except at zero stagger. Making an allowance of 1° or 2° at zero camber will bring the values of optimum incidence closer together, though it would still not give full agreement. Nevertheless some of the disagreement may be as much a criticism of the deviation rules as of the ideas presented in this paper.

On the whole, then, it is considered that the curves in Figs.3, 4 and 5 lend support to the theoretical concept being advanced. It is not doubted that closer correlation could be obtained by introducing further empirical factors such as has been done in Refs.3 and 8 for example. However the express object of this work is to minimise empiricism, and with the introduction of only one empirical factor (i.e., i_a) it may be concluded that this is a satisfactory basis for analysing the optimum incidence of the C4 profile on circular arc camber lines.

3.2 Parabolic arc camber lines (a/c = 40%)

The C4 base profile on P40 camber lines is the second large family of compressor blades which needs examination. Unfortunately the reversed flow cascade (i.e., the P60 camber line at positive stagger) is of no practical significance and the deviation for this blade form is largely unknown. A linear interpolation of m with a/c is often adopted, and hence it was decided to follow a linear extrapolation from the known values at $a/c = 40\%$ and 50% to $a/c = 60\%$. This gave $m = 0.29$, which was again assumed to be independent of stagger over the range considered (see Fig.2). If the same value of i_a applies to both the C50 and P40 camber lines

$$i_{opt} = 6.5° - 0.29\,\theta\frac{s}{c} \text{ (for P40)}\,. \qquad \ldots (9)$$

The theoretical values were compared with the test results on this camber line given in Ref.9. Because the tests were carried out at wide incidence intervals (10° intervals) it was found difficult to estimate the experimental value of i_{opt} accurately. For this reason the test values of the lift/drag ratio have been plotted against inlet angle for representative cascades having pitch/chord ratios of 0.75 and 1.0 in Figs.6 and 7 respectively. Several curves can be drawn through the points, but the author has sketched in what he thinks would be a generally accepted

curve./

curve. Also marked on the curves are the theoretical inlet angles for the maximum. Quite good agreement is obtained. The tendency follows that noted for the C50 camber line - agreement is good at the higher cambers, but experimental maxima tend to occur at higher inlet angles (or incidences) than calculated at the lower, particularly at zero, cambers. Even so agreement at zero camber and unity pitch/chord ratio is fair.

Again better agreement could possibly be obtained by modifying the constants in equation (9) or by introducing new empirical factors but this step is not taken for the reasons already given.

3.3 Parabolic arc camber lines (variable a/c)

As a final check a comparison is made in Fig.8 of the theoretical and experimental variations of the optimum incidence with position of maximum camber. A constant value of $i_a = 6.5°$ was assumed. It will be seen that the curve reproduced from Ref.10 shows a greater experimental variation than is calculated. This curve corresponds to a Mach number of 0.4 and a Reynolds number of approximately 1.4×10^5. A revised curve for a Mach number of 0.5 and a Reynolds number of 1.75×10^5, also calculated from the test results of Ref.10, shows a much closer agreement. The differences in the two experimental curves is apparently due to some peculiar viscous effect - detail plottings are given in Fig.9 to show this. The curve for the higher Reynolds number would probably be the relevant comparison in this case, corresponding with the Reynolds number of 2.2×10^5 for the test results given in Figs.6 and 7. It is considered that the correlation between the calculated and experimental results is good.

It is particularly interesting to note that the lower operating incidence always associated with blades having a forward position of the maximum camber is fully explained by the general equation (5). It shows that blade profiles associated with a low deviation, and hence a high deviation in reversed flow, must also be associated with a low optimum incidence.

4. Discussion

It is considered that the collected evidence given in Sections 3.1, 3.2 and 3.3 is sufficient to substantiate the principle of reversed flow set out in Section 2.2 for estimating the optimum incidence for any cascade at low speeds. There can be little doubt that the proposed method has more fundamental significance than the methods suggested in Refs.3 and 8, and therefore can be used for unconventional cascades with more reliance. As a practical expedient some adjustment can be made to the constant additive incidence (i_a). Thus for low cambered blades it can be arbitrarily increased a few degrees. More particularly in accordance with Section 2.2 one can increase i_a by two or three degrees for the high speed sections C3 and C7 or for two arc sections to bring the calculated values of the optimum incidence into line with the experimental values. Fig.10 shows a comparison of the C4, C7 and C3 profiles. There is surprisingly little difference in optimum incidence for this substantial change in thickness distribution, though the stall behaviour is different, as discussed in Ref.1. Furthermore the low speed operating incidence may have to be modified for high speed operation as at present.

It may also be observed at this stage that the optimum incidence may not be the ideal incidence for design because of its proximity to stall in certain instances. The choice of design incidence in relation to nominal and optimum incidences depends on the position of the section in the blade and the blade in the compressor and is therefore outside the scope of this paper. For the average cascade they would be virtually co-incident however.

The analysis presented in this report has concentrated on compressor cascades, largely because it has been motivated by compressor interests. The general principles should apply to both compressor and turbine cascades. However the range of turbine cascades is so much larger that considerable variations in pressure distribution can be expected from the same profile. It follows that i_a will not be a constant, with the consequent complication in the analysis. Lack of systematic series of turbine profiles also introduces practical difficulties. However it is thought that a re-examination of turbine operating incidences may prove profitable and will be undertaken.

5. Conclusions

Based on a reversed flow hypothesis it has been shown that the optimum incidence for any cascade will be given by

$$i_{opt} = i_a - \delta\left[(-\zeta), \left(1 - \frac{a}{c}\right)\right]. \qquad \dots (5)$$

In this expression i_a has a physical significance, being the additional incidence above that giving the stagnation point at the leading edge. It has a value of about 6.5° for normal profiles.

Comparison has been made with test results from the C4 profile on C50 and P40 camber lines. Agreement is considered good over the practical range of cambers, though discrepancies appear at very low cambers. The agreement is considered good enough to warrant the use of equation (5) for estimating the operating incidence for all unusual cascades.

Notation

a	position of maximum camber from the leading edge
c	blade chord
i	incidence
i_{opt}	optimum incidence (i.e., for maximum lift/drag)
i_a	additive incidence (see Section 2.2)
M_n	Mach number
m	constant in deviation rule (see Fig.2)
R_n	Reynolds number
s	blade pitch
v_n^*	induced velocity perpendicular to chord at L.E.
L.E.	leading edge
T.E.	trailing edge
α	fluid angle measured from axial direction
α_1	fluid inlet angle
α_2	fluid outlet angle
β	blade angle
δ	deviation
θ	camber angle
ζ	stagger (positive for turbine cascades negative for compressor cascades).

References/

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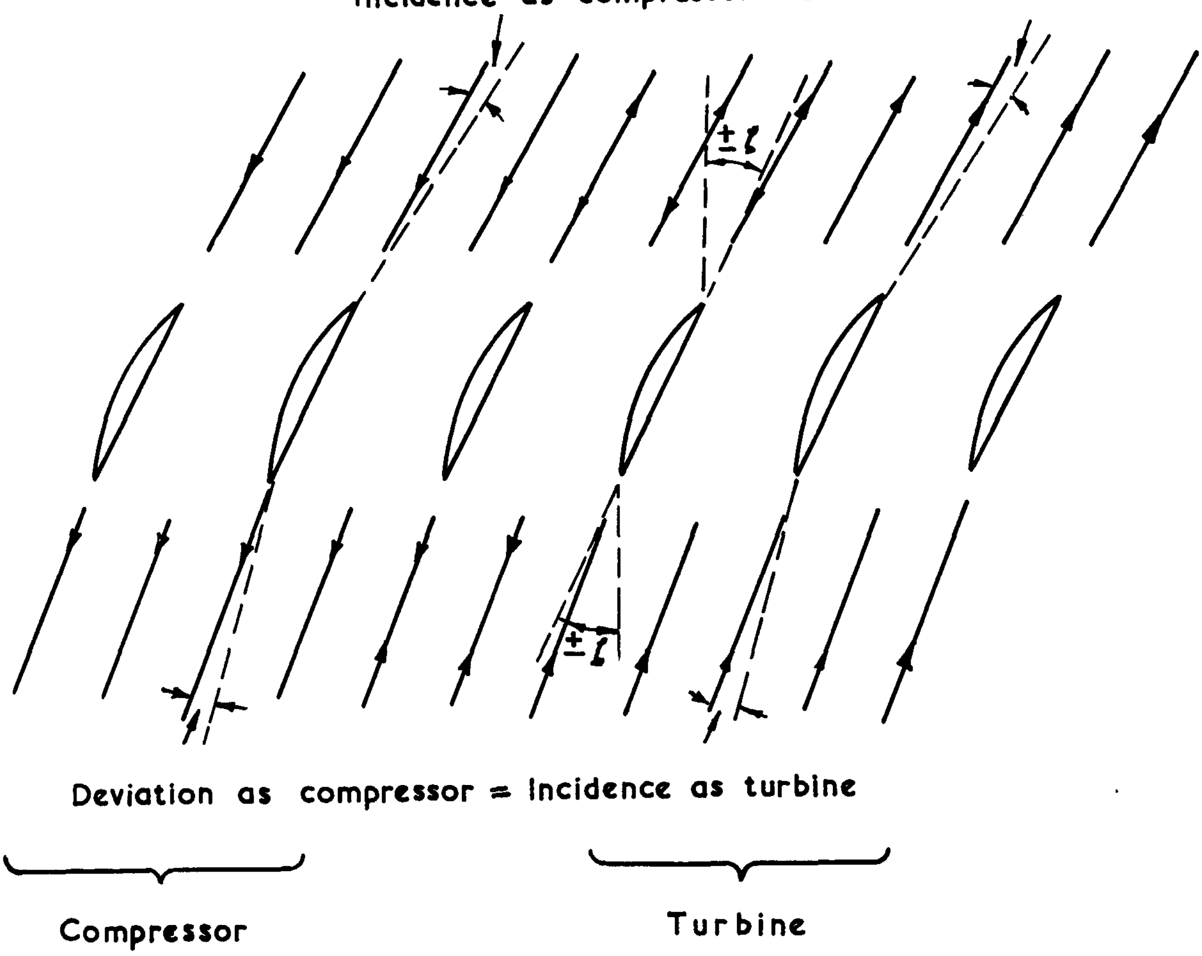
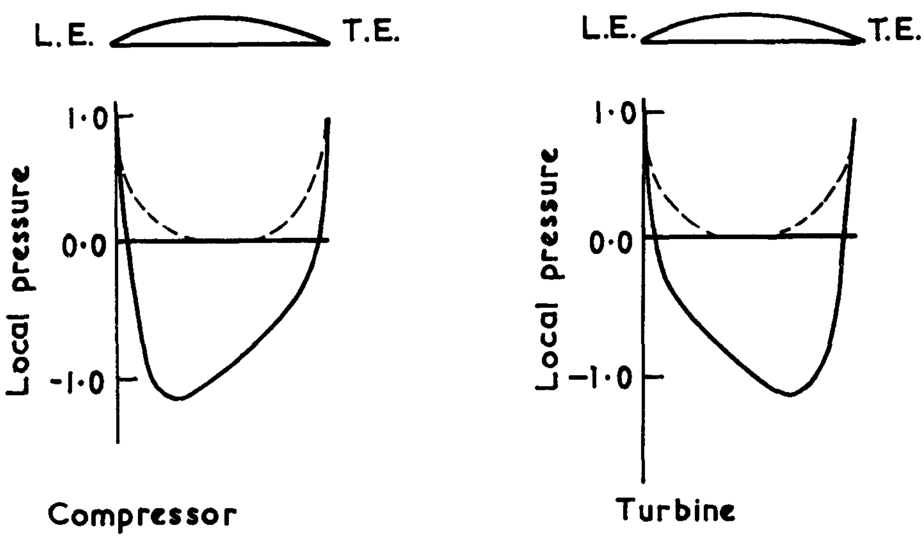
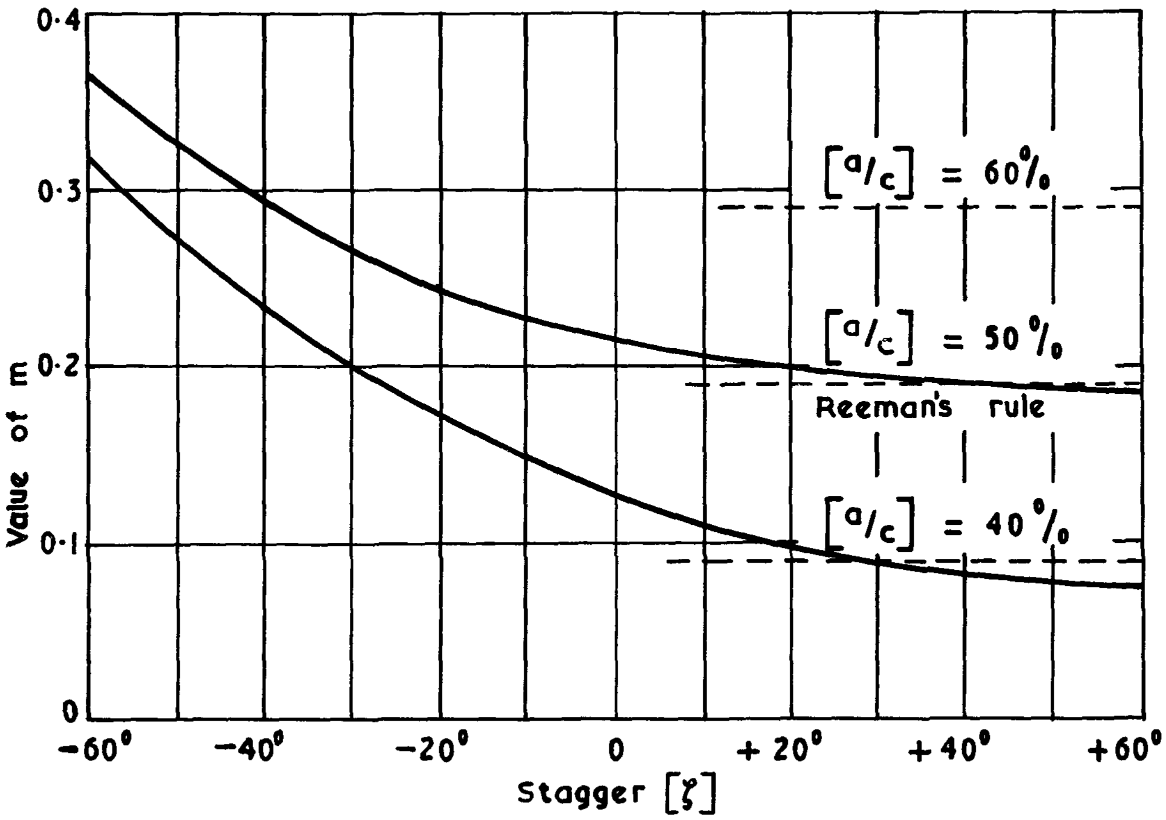
FIG. 1.





Potential flow past a cascade of aerofoils which are symmetrical about the mid-chord position and have sharp leading and trailing edges.

FIG. 2.



——— Values given in Ref 6.

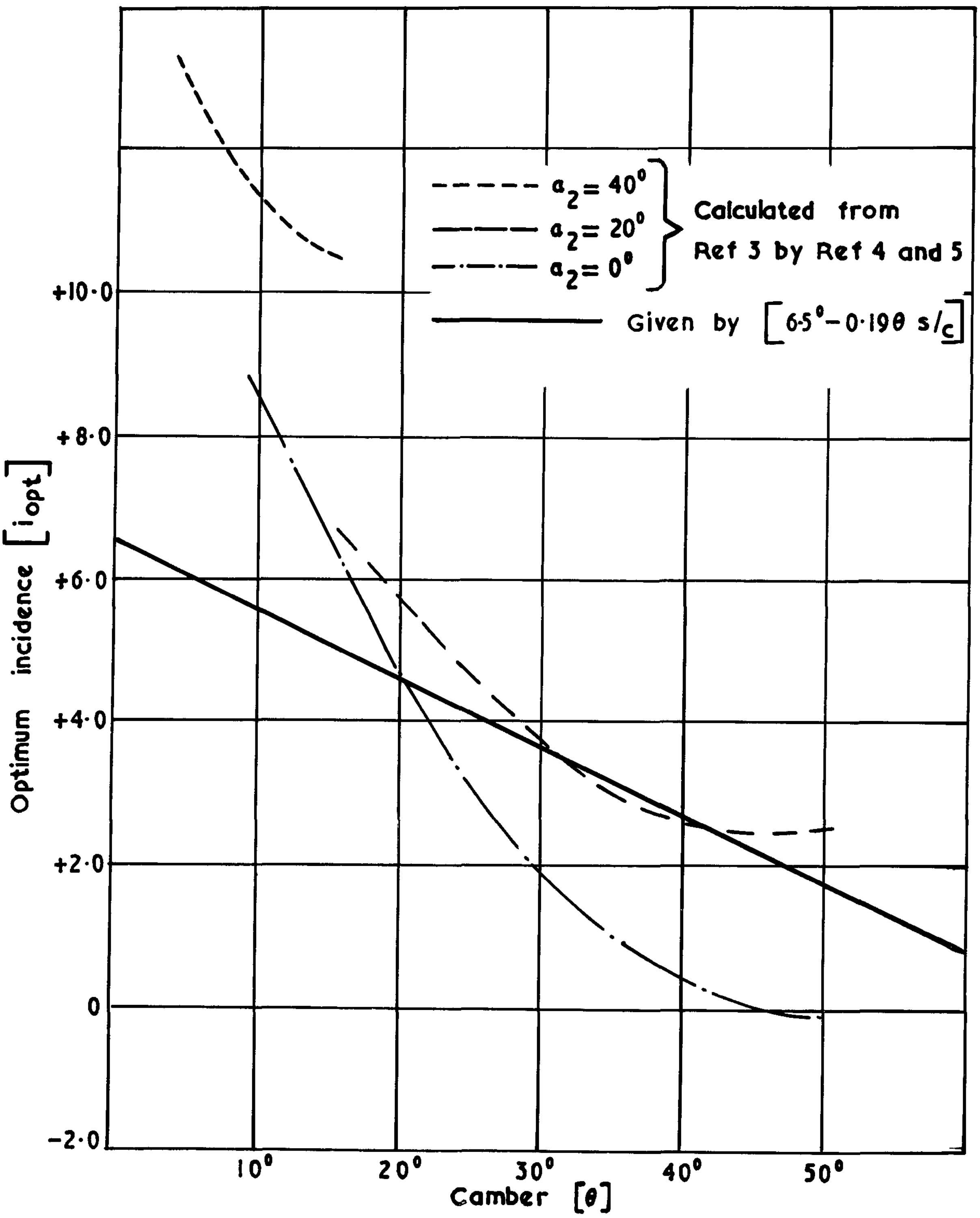
– – – – – Constant values used in analysis over range shown.

Note. $\delta = m\ \theta \sqrt{s/c}$ for compressors

$\delta = m\ \theta\ s/c$ for turbines

Deviation rule

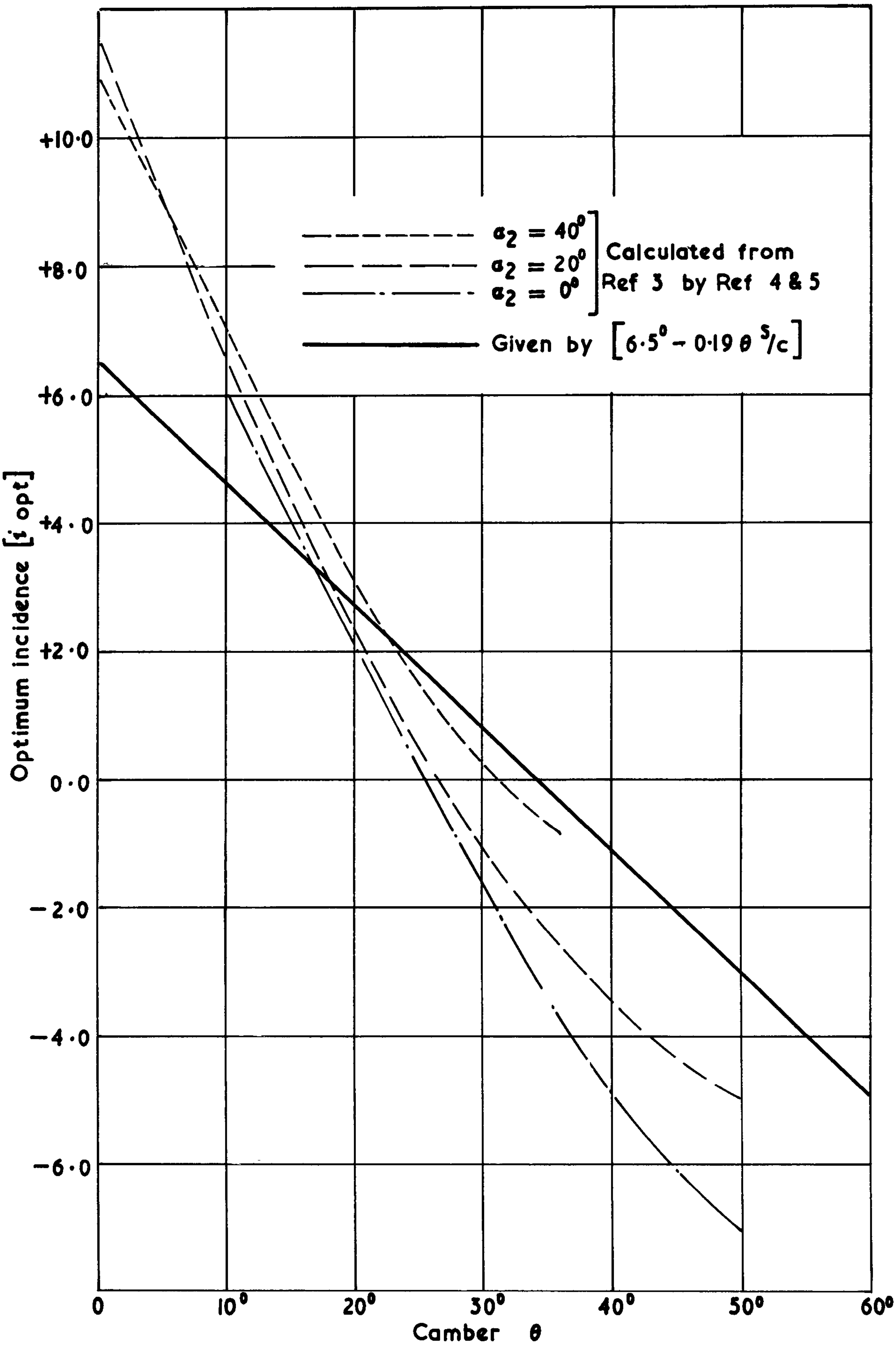
FIG. 3.



Comparison of the optimum incidence for the C4 base profile on circular arc camber lines calculated by different methods.

[Pitch/ chord = 0·5]

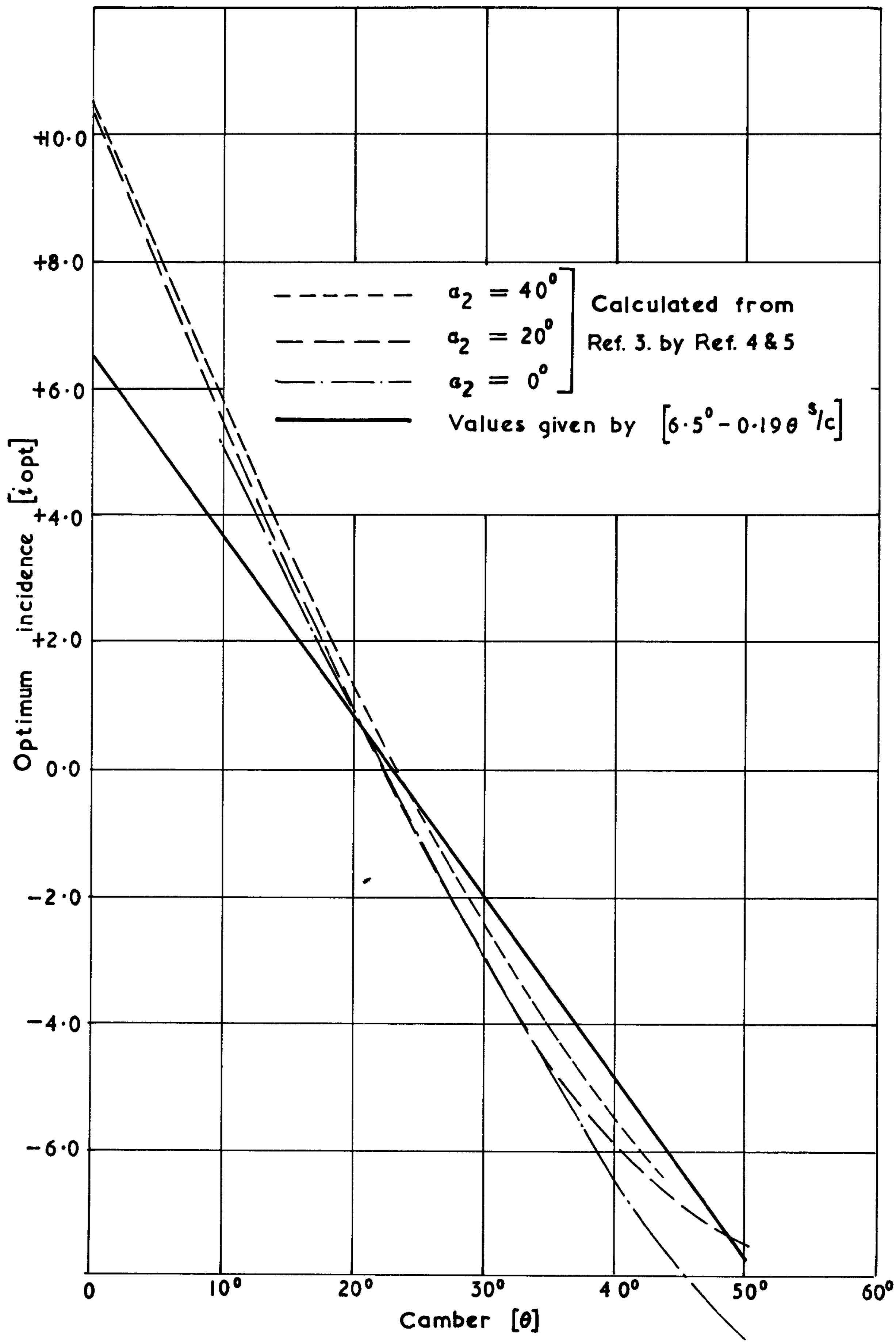
FIG. 4.



Comparison of the optimum incidence for the C_4 base profile on circular arc camber lines calculated by different methods.

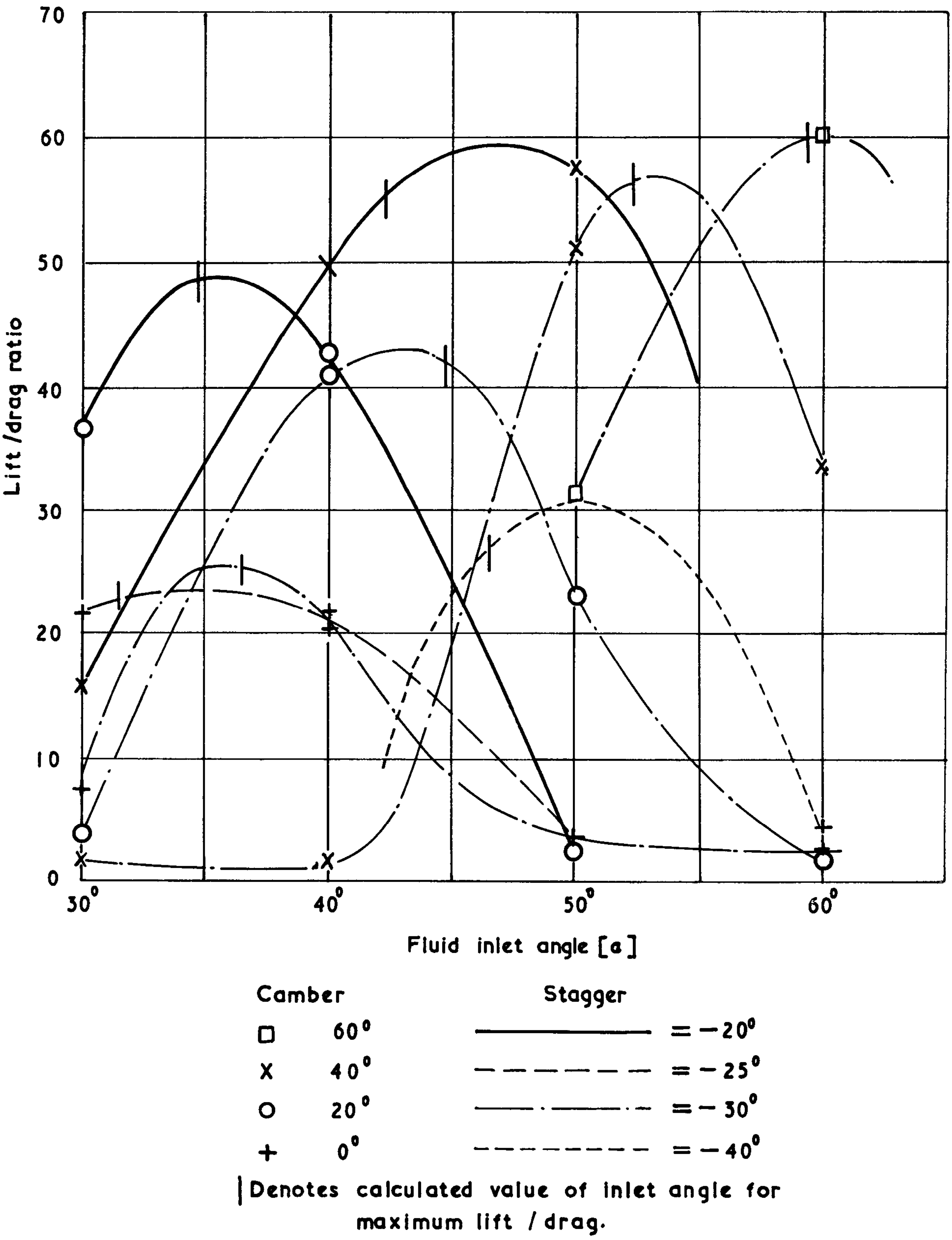
[Pitch / chord = 1·0]

FIG. 5



Comparison of the optimum incidence for the C4 base profile on circular arc camber lines calculated by different methods.
[pitch / chord = 1·5]

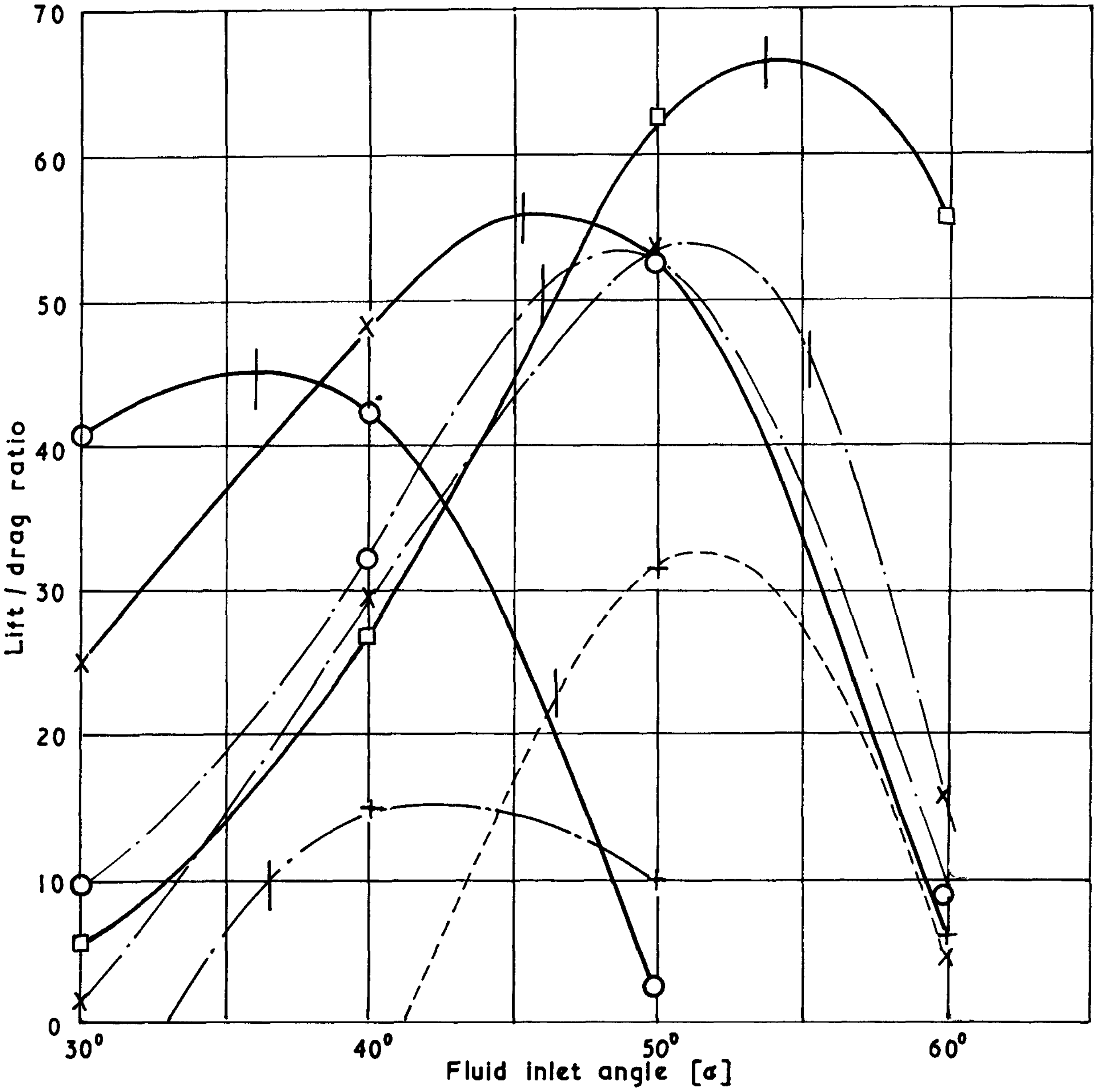
FIG. 6.



Lift /drag ratios for cascades having a pitch/chord ratio of 1·0 showing comparison of measured and calculated inlet angles at which maximum values occur. All curves relate to C4 base profile on P40 camber line. Test results from Ref 9.

$[R_n = 2{\cdot}2 \times 10^5]$

FIG. 7.

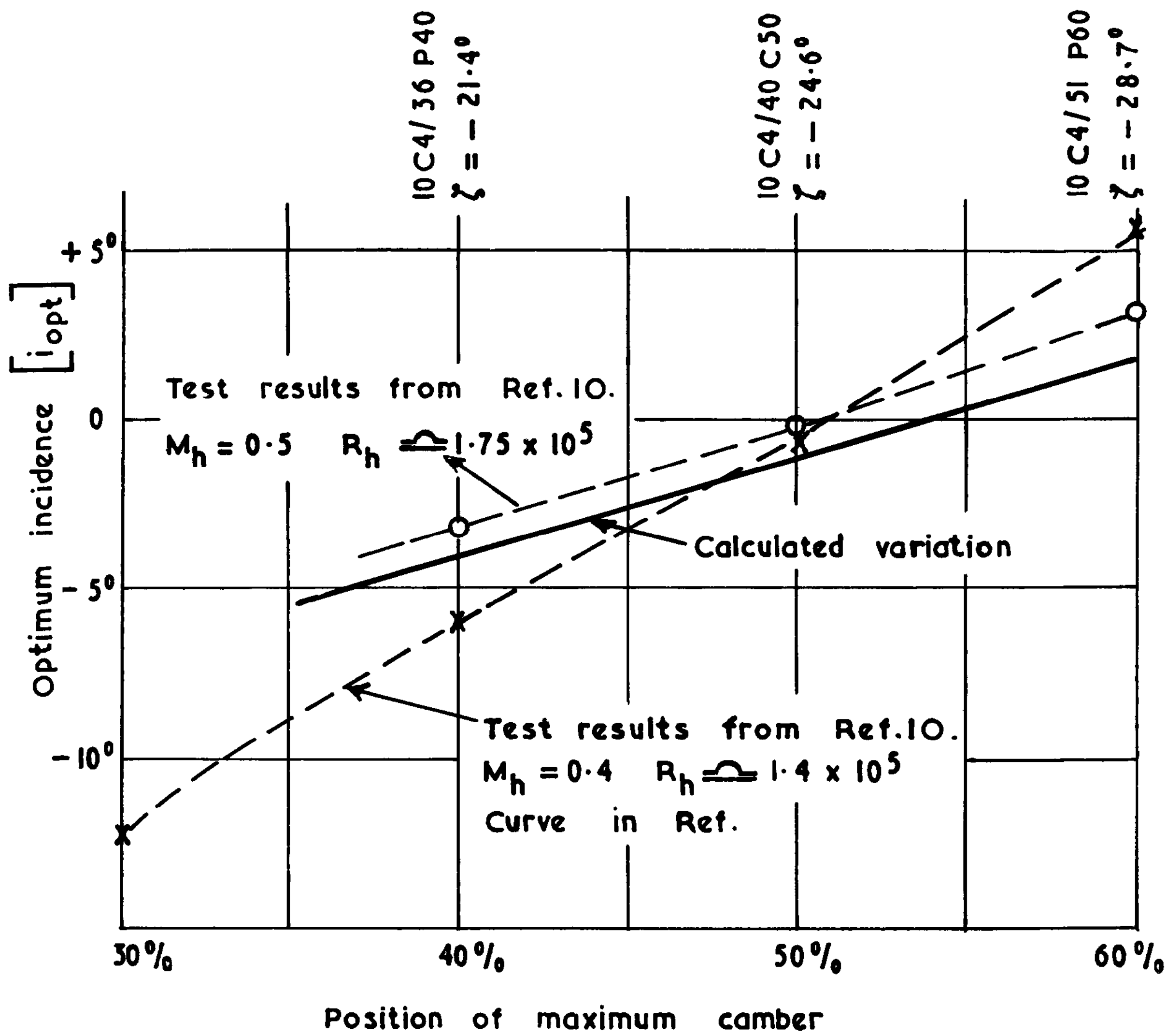


Camber		Stagger	
□	60°	————————	= − 20°
×	40°	—·—·—	= − 30°
○	20°	- - - - -	= − 40°
+	0°		

| Denotes calculated value of inlet angle for maximum lift / drag.

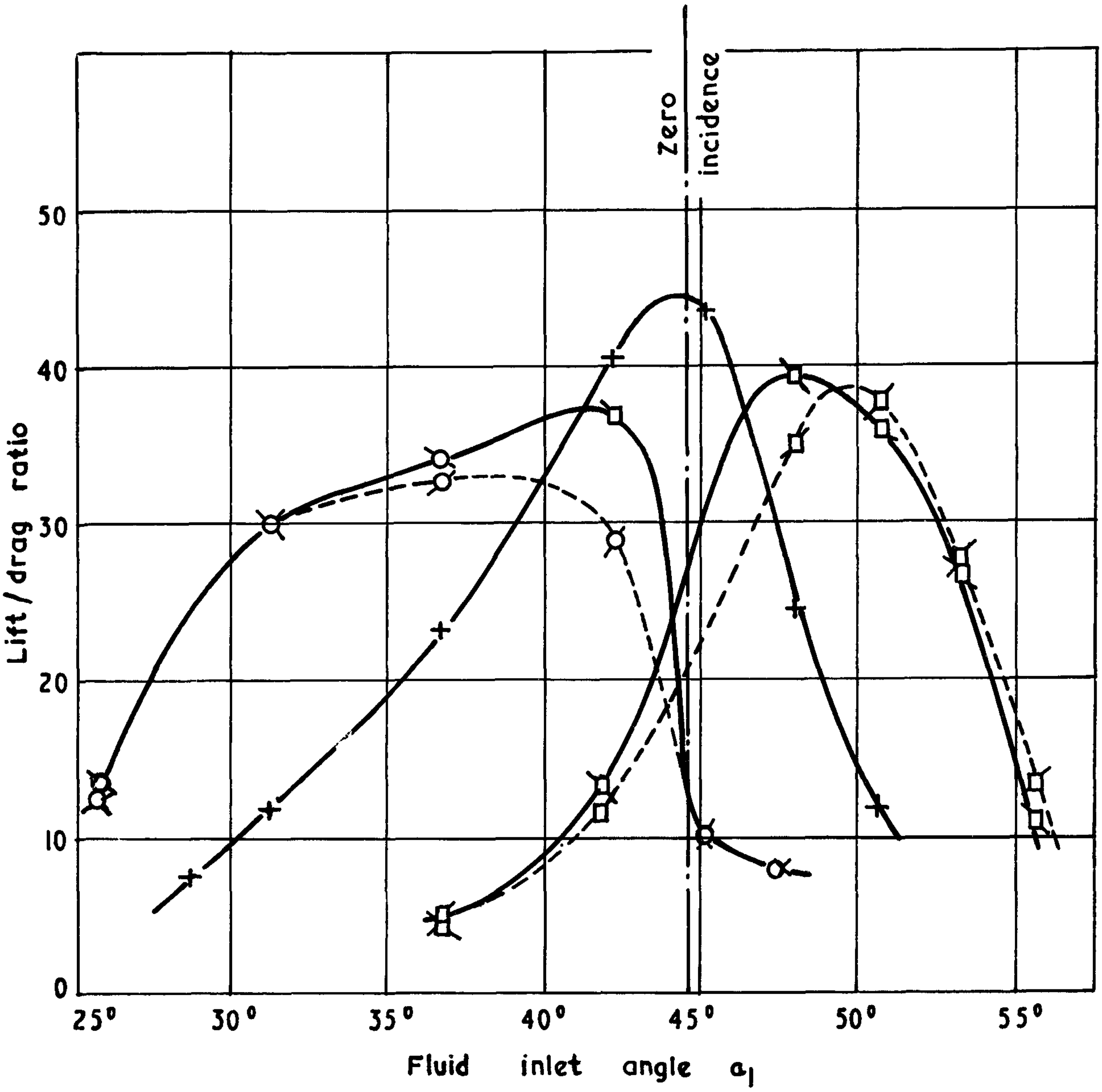
Lift/drag ratios for cascades having a pitch/chord ratio of 0·75 showing comparison of measured and calculated inlet angles at which maximum values occur. All curves relate to C4 base profile on P40 camber line. Test results from Ref. 9. [Rn 2·2 x 10^5]

FIG. 8.



Comparison of theoretical and experimental variation of optimum incidence with position of maximum camber. [Test results from Ref. 10.]

FIG. 9.



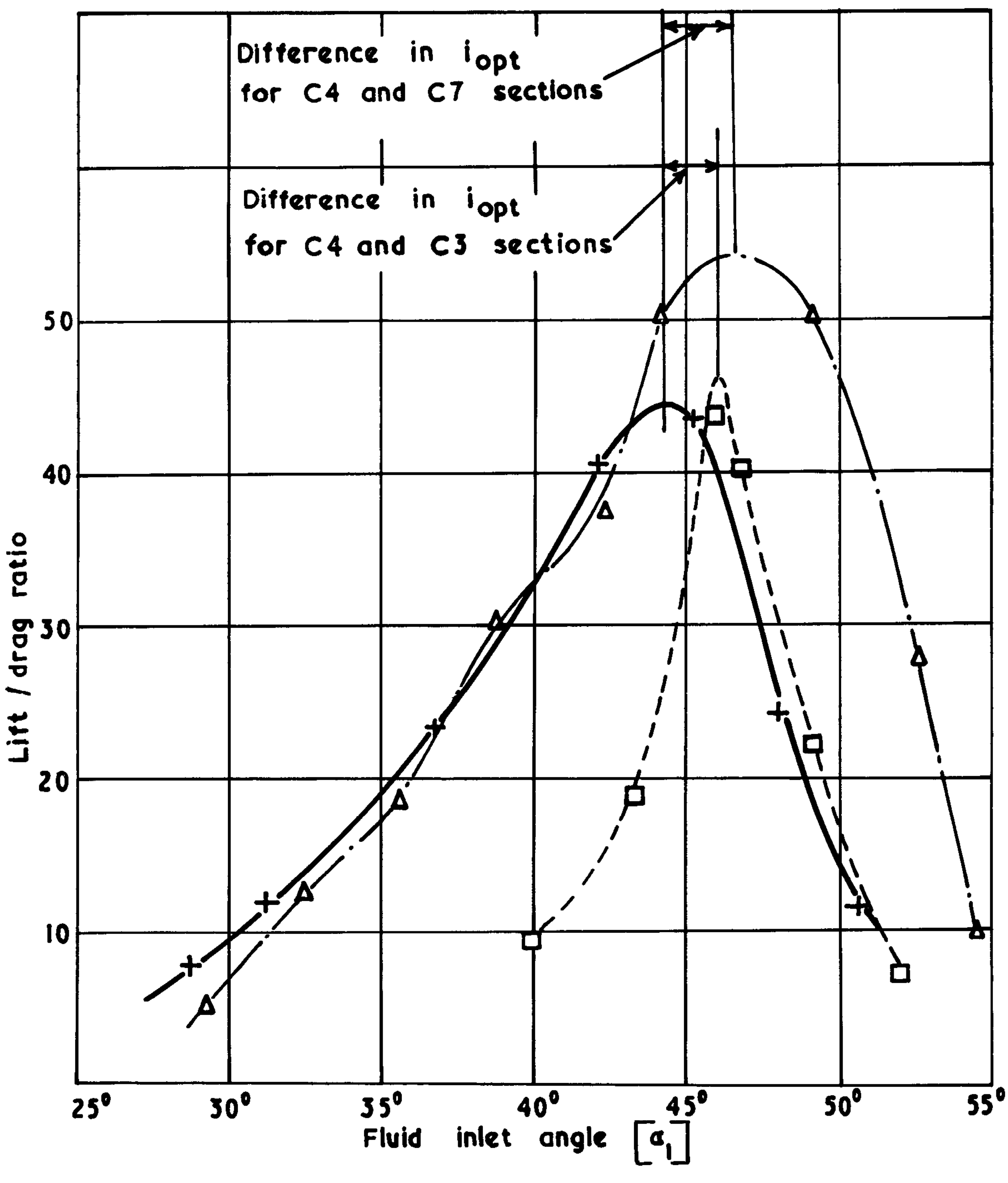
10 C4/51 P60 $\zeta = -28 \cdot 7°$ $s/c = 1 \cdot 0$ { – – □ – – $M_h = 0 \cdot 4$ $R_h = 1 \cdot 4 \times 10^5$
—□— $M_h = 0 \cdot 5$ $R_h = 1 \cdot 75 \times 10^5$

10 C4/40 P50 $\zeta = -24 \cdot 6°$ $s/c = 1 \cdot 0$ —+— $M_h = 0 \cdot 5$ $R_h = 1 \cdot 75 \times 10^5$

10 C4/36 P40 $\zeta = -24 \cdot 1°$ $s/c = 1 \cdot 0$ { —○— $M_h = 0 \cdot 5$ $R_h = 1 \cdot 75 \times 10^5$
– – ○ – – $M_h = 0 \cdot 4$ $R_h = 1 \cdot 4 \times 10^5$

Detail variation of lift / drag ratios used in deriving test curves in Fig. 8.

FIG. 10.



+—+— C4
Δ·—Δ·— C7
□--□-- C3

Camber = 40°
Stagger = 24·6°
Pitch/chord = 1·0°

$M_n = 0{\cdot}5 \quad R_n \simeq 1{\cdot}75 \times 10^5$

Effect of blade thickness distribution on optimum incidence.

[Results from Ref. 1]

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